NONLOCAL REGULARITY FOR THE HEAT FRACTIONAL PROBLEM: APPLICATION TO NONLOCAL PROBLEM WITH NONLOCAL GRADIENT TERM

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Abstract: In this talk we give new regularity results for the heat fractional problem in a natural fractional Sobolev spaces.
More precisely, we consider the following problem

$$\begin{cases} u_t + (-\Delta)^s u = f & \text{in } \Omega_T = \Omega \times (0,T), \\ u(x,t) = 0 & \text{in } (\mathbb{R}^N \setminus \Omega) \times (0,T), \\ u(x,0) = u_0(x) & \text{in } \Omega, \end{cases}$$

where Ω is a bounded regular domain, $f \in L^m(\Omega_T)$ and $u_0 \in L^\sigma(\Omega)$ where $m, \sigma \geq 1$.
According to the value of m, we get the regularity of the "***fractional gradient***" of the solution u.
As a direct application, we will prove an existence result for a fractional Kardar-Parisi-Zhang problem with variant form of the fractional gradient.
A simple one will be the following

$$\begin{cases} u_t + (-\Delta)^s u = \mathbb{D}_s^2(u) + f(x,t) & \text{in } \Omega_T, \\ u(x,t) = 0 & \text{in } (\mathbb{R}^N \setminus \Omega) \times (0,T), \\ u(x,0) = 0 & \text{in } \Omega, \end{cases}$$

where f belongs to a suitable Lebesgue space, $\mathbb{D}_s^2$ is a nonlocal "gradient square" term given by

$$\mathbb{D}_s^2(u)(x,t) = \frac{a_{N,s}}{2} \int_{\mathbb{R}^N} \frac{|u(x,t) - u(y,t)|^2}{|x-y|^{N+2s}} dy\,.$$

According to the value of m, we show existence and non-existence results.
We also obtain existence results for related problems involving different nonlocal **gradient** terms.

The talk is a part of the following paper:

B. Abdellaoui, S. Atmani, K. Biroud, E.-H. Laamri: *Global fractional regularity for the fractional heat problem, application to existence result for nonlocal KPZ equation.*