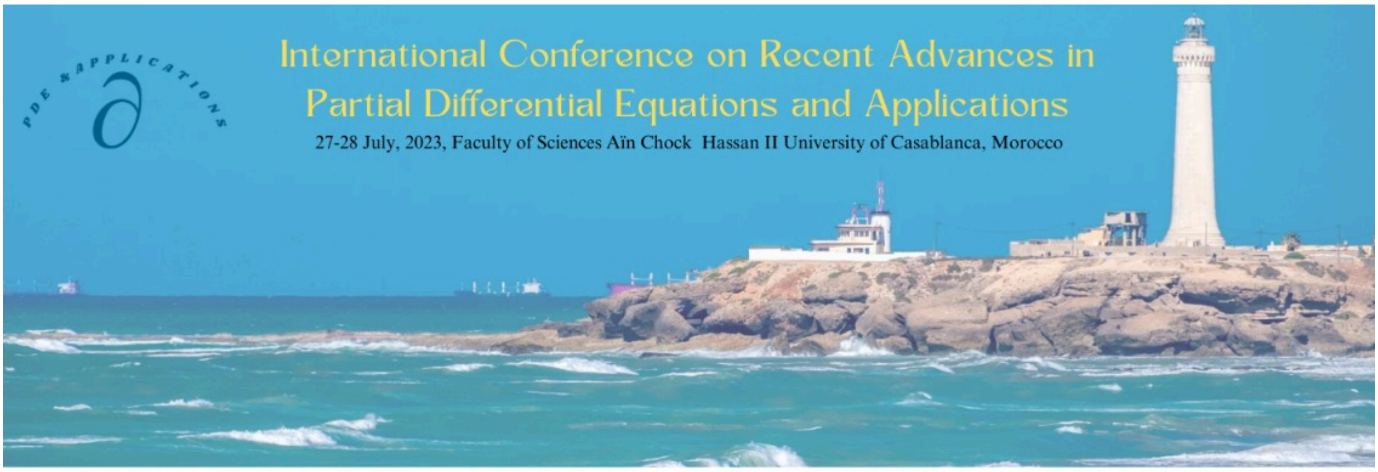
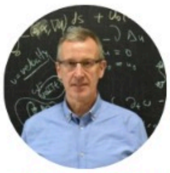


Keynote Speakers


Enrique Zuazua, Germany


Raffaella Servadei, Italy


Jean Rodolphe Roche, France


Khadija Niri, Morocco


Giovanni Molica Bisci, Italy


Boumediene Abdellaoui, Algeria

Topics :

Homogenization and multiscale analysis
Nonlocal PDEs
Hyperbolic conservation laws
Fluid dynamics
PDEs in biological and complex systems
Computational approaches to PDEs
Applications of PDEs in the sciences
Nonlinear Dynamics
Stochastic PDEs
PDE systems
Geometric PDEs and optimal transport
Control theory and Applications
Variational Calculus
Evolution Equations
PDEs in Image processing

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Karim Rhofir (ENSA Khouribga, Université Sultan My Slimane, Morocco)

WELCOME MESSAGE

1

On behalf of the Organizing Committee, we would like to extend a warm welcome to all the participants of the International Conference on Recent Advances in Partial Differential Equations and Applications (ICRAPDEA'23) held on 27-28, 2023 in Casablanca, Morocco.

This scientific event organized by Hassan II University and Faculty of Sciences Ain Chock, Casablanca will provide a remarkable opportunity for national and international academic communities to address new challenges, share their experiences and discuss future research directions in the field of partial differential equations. The technical program will include plenary and regular technical sessions face to face mode and exceptionally in hybrid mode for some participants.

All accepted papers after the peer-review process, will be published as chapters in the journal of Mathematical Modeling and Computing (DOI: 10.23939/mmc, Indexed by Scopus (ISSN: 2312-9794) and Moroccan Journal of Pure and Applied Analysis.

We would like to thank all members of different committees for their efforts before and during the conference and all members of the Technical Program Committee for their hard work in providing reviews in a timely manner. Special thanks also go to all authors for their valuable contributions since ICRAPDEA'23 would not be possible without their contributions.

We are also grateful to all our partners and sponsors, especially Hassan II University and the Faculty of Sciences Ain-Chock, the CNRST, Lorraine university, Institut Elie Cartan etc ….

We hope you enjoy your time with us and we look forward to meeting you all in the next edition of the ICRAPDEA conference.

ICRAPDEA'23 Organizing Committee

SCIENTIFIC COMMITTEE

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NONLOCAL REGULARITY FOR THE HEAT FRACTIONAL PROBLEM: APPLICATION TO NONLOCAL PROBLEM WITH NONLOCAL GRADIENT TERM

Boumediene ABDELLAOUI
Laboratoire d'Analyse Non linéaire et Mathématiques Appliquées
Département de Mathématiques, Université Abou Bakr Belkaïd, Tlemcen
Tlemcen 13000, Algérie

Abstract: In this talk we give new regularity results for the heat fractional problem in a natural fractional Sobolev spaces.
More precisely, we consider the following problem

$$\begin{cases} u_t + (-\Delta)^s u = f & \text{in } \Omega_T = \Omega \times (0,T), \\ u(x,t) = 0 & \text{in } (\mathbb{R}^N \setminus \Omega) \times (0,T), \\ u(x,0) = u_0(x) & \text{in } \Omega, \end{cases}$$

where Ω is a bounded regular domain, $f \in L^m(\Omega_T)$ and $u_0 \in L^\sigma(\Omega)$ where $m, \sigma \geq 1$.
According to the value of m, we get the regularity of the "***fractional gradient***" of the solution u.
As a direct application, we will prove an existence result for a fractional Kardar-Parisi-Zhang problem with variant form of the fractional gradient.
A simple one will be the following

$$\begin{cases} u_t + (-\Delta)^s u = \mathbb{D}_s^2(u) + f(x,t) & \text{in } \Omega_T, \\ u(x,t) = 0 & \text{in } (\mathbb{R}^N \setminus \Omega) \times (0,T), \\ u(x,0) = 0 & \text{in } \Omega, \end{cases}$$

where f belongs to a suitable Lebesgue space, $\mathbb{D}_s^2$ is a nonlocal "gradient square" term given by

$$\mathbb{D}_s^2(u)(x,t) = \frac{a_{N,s}}{2} \int_{\mathbb{R}^N} \frac{|u(x,t) - u(y,t)|^2}{|x-y|^{N+2s}} dy\,.$$

According to the value of m, we show existence and non-existence results.
We also obtain existence results for related problems involving different nonlocal **gradient** terms.

The talk is a part of the following paper:

B. Abdellaoui, S. Atmani, K. Biroud, E.-H. Laamri: *Global fractional regularity for the fractional heat problem, application to existence result for nonlocal KPZ equation.*

1

Title: **Quasilinear elliptic equations on Finsler manifolds**

Giovanni Molica Bisci (Università di Urbino Carlo Bo, Pesaro e Urbino, Italy)

Abstract: The theory of Sobolev spaces on complete Riemannian manifolds is well understood and widely applied into the study of various elliptic problems. Although Finsler geometry is a natural extension of Riemannian geometry, very little is known about Sobolev spaces on non-compact Finsler manifolds. Motivated by this wide interest in the literature, the leading purpose of this talk is to present some recent results on non-compact Randers spaces and their applications to quasilinear elliptic equations. The main approach is based on novel abstract Sobolev embedding results as well as on some variational and topological methods developed in the recent book Nonlinear Problems with Lack of Compactness, De Gruyter Series in Nonlinear Analysis and Applications 36 (2021), co-authored with P. Pucci.

Title: **Le Smart Power Mathématique dans la gestion des grandes épidémies de l'histoire**

Khadija Niri (Faculté des Sciences Ain Chock, Université Hassan II de Casablanca)

Abstract: Les épidémies ont toujours été une menace pour la santé publique et ont souvent eu un impact considérable sur la société et l'histoire de l'humanité. Au fil des siècles, les mathématiciens ont cherché à comprendre les mécanismes de la propagation des maladies infectieuses et ont développé des modèles mathématiques pour décrire et prévoir les épidémies. Dans cet exposé, on essaiera de présenter l'évolution de ces modèles et le développement de certains concepts associés comme le R0 la taille finale et l'équilibre endémique. On illustrera par des exemples comment la compréhension, l'analyse mathématique et l'étude numérique de ces derniers sont devenus essentiels pour prédire la propagation des maladies et pour concevoir des stratégies ef icaces de la santé publique.

Title: **A domain decomposition method for the numerical approximation of the non-negative solution of non-linear parabolic equations.**

Jean Rodolphe Roche (Université de Lorraine)

Abstract:

The aim of this talk is to present a numerical method to compute a numerical approximation of a non-negative solution of semi-linear parabolic equations. It concerns the case of solutions with or without blow-ups. For this purpose, we developpe two different algorithms combining Crank-Nicholson time discretization schema, finite elements approximation, Newton method, and domain decomposition technics.

In the case of the blow-up, we will give an estimation of the maximal time existence of the numerical solution. That estimation is a precise approximation of the maximal time existence of the analytical solution. We will also adapt the para-real algorithm to the non-linear problem. In all cases, the simulations converge and illustrate the performance of the algorithms studied and the coherence of the results with the theory.

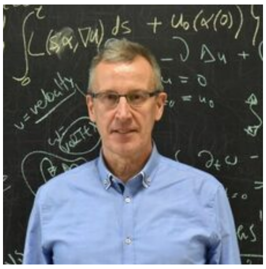
This is joint work with: **Nahed Naceur, Nour Eddine Alaa and Moez Khenissi.**

Title: **On fractional equations with critical growth**

Rafaella Servadei (Università degli Studi di Urbino Carlo Bo, Italy)

Abstract:

Fractional and nonlocal operators appear in concrete applications in many different fields. This is one of the reason why, recently, nonlocal fractional problems are widely studied in the literature. Critical problems are particularly relevant for their relations with many applications where a lack of compactness occurs. Aim of this talk is to discuss some recent results about existence and multiplicity of solutions for fractional nonlocal equations with critical growth assumptions on the nonlinear term.



Enrique ZUAZUA

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[1] FriedrichAlexanderUniversität ErlangenNürnberg - Alexander von Humboldt Professorship, Germany
[2] Fundación Deusto, Bilbao
[3] Universidad Autónoma de Madrid

Title : Control and Machine Learning

Abstract: Control, or Cybernetics, according to the term coined first by Ampère and then popularized through the works of Wiener, is the science of control and communication in animals and the machines. The motivation goes back to ancient times: machines that automatically carry out man's work so that he can be freer and more efficient. The goals of Control are therefore, to a large extent, those of the modern discipline of Machine Learning.

In this talk we shall present some fundamental mathematical notions and techniques in Control that have strongly influenced the emergence and development of the field of Machine Learning. We will do it analyzing Neural Ordinary Differential Equations (NODEs) from a control theoretical perspective to address some of the main challenges in Machine Learning:Supervised Learning and Universal Approximation.

We adopt the perspective of the simultaneous or ensemble control, according to which, each item to be classified or learned corresponds to a different initial datum for the Cauchy problem of a NODE. The challenge is then to control the ensemble of solutions to the corresponding targets by means of one sole control. We present a genuinely nonlinear and constructive method, allowing to show that such an ambitious goal can be achieved, estimating the complexity of the control.

This property is rarely fulfilled by the classical dynamical systems in Mechanics and, as we shall see, it is intimately related to the very nonlinear nature of the activation functions governing the NODEs under consideration. It allows deforming half of the phase space while the other half remains invariant, a property that classical models in mechanics do not fulfill. Analyzing the natural consequence of Universal Approximation, we shall also establish the link with optimal transport.

We shall also illustrate how, classical concepts and tools of Control Theory, such as the “turnpike property” allow the training of Neural Networks ina more stable and robust manner.

Joint work with **Borjan Geshkovski**, **Carlos Esteve**, **Domènec Ruiz-Balet** and **Dario Pighin**.

MINISYMPOSIA

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MS: Mini-symposium	
MS1: Control theory & Inverse problem	Lahcen MANIAR, FSSM, Université Cadi Ayyad, Marrakech, Morocco
MS2: Nonlocal PDEs and its applications	Title : *On some nonlinear fractional systems arising in the biological* Elhoussine AZROUL, FSDM, Université Sidi Mohamed Ben Abdellah, Fès, Morocco
MS3: PDEs in Biological and Complex Systems	Title 1: *Partial differential equations applied to infectious diseases and economics.* Khalid HATTAF, CRMEF of Casablanca, Morocco Title 2: *Global proprieties of a spatial heterogeneous delayed epidemic model with partial susceptible protection and nonlocal disperse*, Abdessamad TRIDANE, Mathematical sciences-(COS), Al Ain, UAE
MS4: Integral transforms of Fourier type	Title 1: *Equivalence of K-Fonctionals and Modulus of Smoothness Constructed by Generalized Jacobi Transforms* Mohamed EL HAMMA, FSAC, Université Hassan II de Casablanca, Morocco Title 2: *Generalized Abilov's Theorem for The Multidimentional Fourier-Bessel Transform* Abdellatif AKHLIDJ, FSAC, Université Hassan II de Casablanca, Morocco
MS5: Elliptic and Parabolic Equations and systems	Azeddine BAALAL, FSAC, Université Hassan II de Casablanca, Morocco El Haj LAAMRI, Institut Élie Cartan, Université de Lorraine, France

MS6: PDEs in Image processing	**Driss MESKINE, EST Essaouira, Université Cadi Ayyad, Morocco** **Title :** ***Sur quelques équations et systèmes de réaction-diffusion et application en imagerie*** **Presented by Fahd KARAMI, EST Essaouira, Université Cadi Ayyad, Morocco**
MS7: Computational approaches to PDEs	**Mostafa OUARIT & Atika RADID, FSAC, Université Hassan II de Casablanca, Morocco**

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PDEs & Applications

International Conference on Recent Advances in Partial Differential Equations and Applications (ICRAPDEA'23)

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Analysis of a continuous tuberculosis model with smoking consideration

Issam KHALOUFI[1], **Abdelfatah KOUIDERE** [1], **Hassan LAARABI** [1], **Mostafa RACHIK** [1]

[1] Laboratory of Analysis Modeling and Simulation, Casablanca, Morocco, 20670.
issam.khaloufi1@gmail.com

Abstract:
In this paper, we propose a mathematical model that simulates the evolution of tuberculosis in time taking into account the division of the population according to the smoking criterion. We also collect some preliminary results, calculate the fundamental reproduction rate and study the stability of the free equilibrium point. In addition, we propose four control strategies. Finally, numerical results produced by Matlab confirmed the theoretical conclusions.

Keywords: Mathematical model ; Optimal control ; Tuberculosis; Smoking ; Contagious virus ; Local stabilty; Dynamic system;; infectious diseases, Stability ; Free equilibrium ; Pontryagin maximum.

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Global Stability of an age-structured HIV infection model with latency and cell-to-cell transmission

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Abstract:

In this work, we propose an age-structured HIV infection model with latency and cell-to-cell transmission. The proposed model formulated by ordinary differential equations (ODEs) and partial differential equations (PDEs). The well posedness and the existence of equilibria are fully established. Moreover, the qualitative properties including uniform persistence, local stability of equilibria as well as the global behavior of the model are rigorously studied.

Keywords: PDEs, HIV infection, age-structure, uniform persistence, stability.

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A fractional model of partial differential equations for HIV-1 infection with highly active antiretroviral therapy

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Abstract:

Highly active antiretroviral therapy (HAART) is a medication regimen used to treat human immunodeficiency virus type 1 (HIV-1). On the other hand, immunological memory is an important characteristic of humoral immunity. In this work, we propose a mathematical model that incorporates spatial effects, immunological memory and general incidence rate to describe the dynamics of HIV-1 infection in the presence of HAART. First, we show that the developed model is mathematically and biologically well-posed. Additionally, we discuss the existence and stability of equilibrium points. We thoroughly analyze the impact of memory and HAART on the dynamical behavior of our suggested model.

Keywords: Diffusion, HIV-1 infection, therapy, humoral immune response, global stability.

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Dynamics of a diffusive business cycle model with two delays and variable depreciation rate

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[2]Equipe de Recherche en Modélisation et Enseignement des Mathématiques (ERMEM), Centre Régional des Métiers de l'Education et de la Formation (CRMEF), 20340 Derb Ghalef, Casablanca, Morocco

Abstract:
The main aim of this work is to analyze the dynamics of a delayed business cycle model described by partial differential equations (PDEs) in order to take into account the depreciation rate of capital stock and the diffusion effect. Firstly, the existence of solutions and the economic equilibrium are carefully studied. Secondly, the local stability and the existence of Hopf bifurcation are established. Finally, some numerical simulations are presented to illustrate the analytical results.

Keywords: Business cycles, depreciation rate, diffusion effect, Hopf bifurcation, stability.

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International Conference on Recent Advances in Partial Differential Equations and Applications (ICRAPDEA'23)

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Global stability of fractional partial differential equations applied to a biological system modeling viral infection

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[2]Equipe de Recherche en Modélisation et Enseignement des Mathématiques (ERMEM), Centre Régional des Métiers de l'Education et de la Formation (CRMEF), 20340 Derb Ghalef, Casablanca, Morocco

Abstract:
In this work, we investigate the global stability of fractional partial differential equations applied to a biological system modeling viral infection. The reaction into the proposed biological system described by the new generalized Hattaf fractional (GHF) derivative. However, the diffusion is modeled by Laplacian operator.

Keywords: Fractional partial differential equation, Hattaf fractional derivative, Diffusion, Global stability.

1 Introduction

Hepatitis B is a viral infection that affects the liver and can cause acute or chronic illness. To understand the dynamics of this infection in a population, several authors have studied the dynamics of viral infections using partial differential equations (PDEs) which enable us to describe how variables change with respect to both time and space, which is not achievable through ordinary differential equations (ODEs) modeling [1, 2]. For this reason, modeling of hepatitis B virus (HBV) infection with PDEs has been the subject of a great deal of research [3, 4, 5].

In some cases, fractional differential equations (FDEs) can be used to model certain aspects of infectious diseases more accurately, allowing phenomena such as long term memory, diffusion processes or long range interactions to be taken into account, which is not the case with PDE modeling.

This is why fractional derivatives have attracted the attention of many researchers. For instance, Ullah et al. [6] proposed a mathematical model for the hepatitis B virus with the Caputo-Fabrizio fractional derivative with non-singular kernel. In [7], Din et al. investigated a mathematical model for the hepatitis B virus with the Atangana-Baleanu Caputo (ABC) derivative. In [8], Bachraoui et al. studied the stability of a fractional reaction-diffusion model, the infection transmission is modeled by Hattaf-Yousfi functional response and the fractional derivative is in the sense of Caputo.

Recently, Hattaf [9] introduced a new generalized definition of the fractional derivative with a non-singular and non-local kernel for Caputo and Riemann-Liouville types, which is used to study the impact of the memory effect on the dynamics of certain dynamical systems in epidemiology and biology. This definition generalizes the most famous fractional derivatives of non-singular kernels existing in the literature [10, 11, 12].

This paper aims to study the fractional order HBV model based on the generalized Hattaf fractional derivative [9]. The remaining part of this paper is organized as follows. In Section 2, model formulation is given. The stability of equilibria of our model is discussed in Section 3. Finally, in Section 4, we give a conclusion for our work.

2 Model formulation

We extend the model given in [5] by using the GHF derivative in order to describe the dynamics of HBV infection under the effects of diffusion and memory. Hence, the model becomes:

$$\begin{cases} \partial_t^{\alpha,\beta} u(x,t) = \lambda - du(x,t) - f(u(x,t), w(x,t), v(x,t)), \\ \partial_t^{\alpha,\beta} w(x,t) = f(u(x,t), w(x,t), v(x,t))v(x,t) - aw(x,t), \\ \partial_t^{\alpha,\beta} v(x,t) = d_v \Delta v + kw(x,t) - mv(x,t), \end{cases} \tag{1}$$

where the general incidence function $f(x,y,v)$ is assumed to be continuously differentiable in the interior of $\mathbb{R}^3_+$ and satisfies the three fundamental hypotheses:

$f(0,y,v) = 0,$ for all $y \geq 0$ and $v \geq 0$

$\frac{\partial f}{\partial x}(x,y,v) > 0,$ for all $x > 0$, $y \geq 0$ and $v \geq 0$

$\frac{\partial f}{\partial y}(x,y,v) \leq 0$ and $\frac{\partial f}{\partial v}(x,y,v) \leq 0,$ for all $x \geq 0$, $y \geq 0$ and $v \geq 0$.

In biological terms, $u(x,t)$, $w(x,t)$ and $v(x,t)$ represent the densities of uninfected cells, infected cells and free virus at location x and time t, respectively. The parameter λ is the recruited rate of uninfected cells, k is the production rate of free virus by infected cells, d, a and m are respectively, the death rates of uninfected cells, infected cells and free virus. d_v is the diffusion coefficient. $\Delta = \sum_{i=1}^{n} \frac{\partial^2}{\partial x_i^2}$ denote the Laplacian operator. The partial operator $\partial_t^{\alpha,\beta}$ which is a simplification of the operator $\partial_{0,w,t}^{\alpha,\beta,\gamma}$ is the partial generalized Hattaf fractional derivative in Caputo sense with $\alpha \in [0, 1)$ and $\beta > 0$ [9], which describes the memory effect.

3 Equilibrium Points and Their Stability

3.1 Equilibrium Points

From the results presented by Hattaf et al. [13], the basic reproduction number of virus in the absence of spatial dependence is given by

$$\mathcal{R}_0 = \tfrac{k}{am} f(\tfrac{\lambda}{d}, 0, 0).$$

By a simple computation and by using Theorem 2.1 [13], we get the following result:

Theorem 3.1 *1. If $\mathcal{R}_0 \leq 1$, then the system 1 has a unique disease-free equilibrium of the form $E_f(\frac{\lambda}{d}, 0, 0)$.*
2. If $\mathcal{R}_0 > 1$, then the system 1 has a unique chronic infection equilibrium of the form $E^*(u^*, w^*, v^*)$ where $u^* \in (\frac{\lambda}{d}, 0)$, $w^* > 0$ and $v^* > 0$.

3.2 Stability of the disease-free equilibrium

In this subsection, we rigorously investigate the stability of E_f.

Theorem 3.2 *The disease-free equilibrium E_f is globally asymptotically stable whenever $\mathcal{R}_0 \leq 1$ and it is unstable otherwise.*

3.3 Stability of the chronic infection equilibrium

Now, we should establish a set of conditions which are sufficient for the global stability of E^* Thus, we give the following further assumption

$$\left(1 - \tfrac{u}{u_0}\right)\left(\tfrac{f(u,w^*,v^*)}{f(u,w,v)} - \tfrac{v}{v^*}\right) < 0, \qquad \text{for all } u, w, v > 0. \tag{H$_1$}$$

The following Theorem establishes the global stability of E^*.

Theorem 3.3 *Assume $\mathcal{R}_0 > 1$ and (H_1) hold, then the chronic infection equilibrium E^* is globally asymptotically stable.*

4 Conclusion

In this paper, we have developed a virus dynamics model with general incidence rate. The global asymptotic stability of the disease-free equilibrium E_f and the chronic infection equilibrium E^* have been established by using suitable Lyapunov functionals and LaSalle invariance principle. We have shown that E_f is globally asymptotically stable if the basic reproduction number satisfies $R_0 \leq 1$. In this case, all positive solutions converge to E_f and the virus is unable to maintain the infection and will go extinct. When $R_0 > 1$, E_f becomes unstable and there occurs the equilibrium E^* which is globally asymptotically stable.

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A generalized diffusive IS-LM business cycle model with delays in gross product and capital stock

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Abstract:

In this work, we propose a diffusive and delayed IS-LM business cycle model with general investment, interest rate and money supply under homogeneous Neumann boundary conditions. The time delays are introduced into gross product and capital stock respectively. We first prove that the model is mathematically and economically well posed. By analyzing the corresponding characteristic equation, the local stability of the economic equilibrium and the existence of Hopf bifurcation are investigated. In the end we show some numerical simulations to verify our theoretical results.

Keywords: IS-LM business cycle, time delays, diffusion, stability, Hopf bifurcation.

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Spatiotemporal dynamics of cancer with oncolytic virotherapy and inhibitors

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Abstract:
In this work, we present a mathematical model with PDEs to treat cancer with two different therapies, oncolytique viruses and MEK inhibitors. We show that our model is biologically and mathematically well-posed through the existence, the non-negativity and the boundedness of solutions. Furthermore, we study the equilibrium points as well as the stability of these equilibria. Finally, we use numerical simulations to illustrate the effect of this combined therapy on tumor cells.

Keywords: PDEs, MEK inhibitors, mathematical modeling, oncolytic virus.

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Global stability of a generalized HBV model with anomalous diffusion and two delays

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Abstract:
This work explores the global dynamics of a generalized partial differential equations (PDEs) model for the infectious disease of hepatitis B virus (HBV), taking into account anomalous spatial diffusion. The proposed model incorporates two time delays and is subjected to non-local Neumann boundary conditions. The diffusion process within the model is mathematically formulated using the fractional Laplacian operator. Furthermore, the stability disease-free equilibrium and the chronic infection equilibrium are analyzed by means of Lyapunov direct method.

Keywords: HBV infection, anomalous diffusion, fractional Laplacian operator, asymptotic stability.

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Sensitivity Analysis of Social Media Addition

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Abstract:

In this paper, we have developed a deterministic mathematical model of social media addiction. Major qualitative analyses such as the balance point without social media dependency (E_0), the endemic equilibrium point (E_*), the basic reproduction number (E_0), were estimated. From the stability analysis, we found that the social media-free equilibrium point (SMAFEP) is only locally asymptotically stable if $E_0 < 1$. The global asymptotic stability of SMAFEP is installed using the Castillo-Chavez theorem. If $E_0 > 1$ the unique endemic equilibrium is also locally asymptotically stable.
Also using Center Manifold theorem, the model exhibits a forward bifurcation at $E_0 = 1$. The sensitivity of the model parameters is determined by defining the normalized forward sensitivity index.

Keywords: Stablity analysis, Social media addiction, Compartmental model, numerical simulation.

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July 27-28, 2023, Faculty of Sciences Casablanca, Hassan II University, Casablanca Morocco

Boundary Controllability For A Degenerate And Singular Wave Equation

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Abstract:

In this work we deal with the boundary controllability of a one-dimensional degenerate and singular wave equation with degeneracy and singularity occurring at the boundary of the spatial domain. Exact boundary controllability is proved in the range of both subcritical and critical potentials and for sufficiently large time, through a boundary controller acting away from the degenerate/singular point. By duality argument, we reduce the problem to an observability estimate for the corresponding adjoint system, which is proved by means of the multiplier method and new Hardy-type inequalities.

Keywords: boundary controllability, observability estimate, degenerate wave equations, singular potential, Hardy inequalities.

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Convergence of FV Scheme for a Parabolic System in Image Processing.

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Abstract:

We analyze a finite volume scheme for a nonlinear reaction-diffusion system used in image processing. First, we establish the existence of a solution to the scheme. Then, by deriving a series of a priori estimates and employing Kolmogorov's compactness criterion, we prove the convergence of the finite volume solution to the weak solution. Our numerical experiments demonstrate the effectiveness of the proposed model in preserving small details, texture, and fine structures when compared to the modified Perona-Malik nonlinear image selective smoothing equation proposed by Catté, Lions, Morel, and Coll [1].

Keywords: Image Processing, Perona-Malik Equation, Finite Volume Method, Convergence, Nonlinear Reaction-diffusion.

1 Introduction

Our work presents a novel nonlinear reaction-diffusion system that effectively enhances coherence. Taking inspiration from the Perona-Malik diffusion process, we have devised an approach that combines its strengths. The equation we propose, incorporating Neumann boundary conditions, is as follows

$$\begin{cases} \dfrac{\partial u}{\partial t} - div\Big(g(|\nabla G_\sigma * u|)\nabla u\Big) + 2\lambda w = 0 & \text{in } \Omega_T := \Omega \times (0,T), \\ \dfrac{\partial w}{\partial t} = \Delta w - (u_0 - u) & \text{in } \Omega_T := \Omega \times (0,T), \\ \langle g(|\nabla G_\sigma * u|)\nabla u, \nu\rangle = 0, \quad \dfrac{\partial w}{\partial \nu} = 0 & \text{on } \Sigma_T := \partial\Omega \times (0,T), \\ u(0) = u_0, \quad w(0) = 0 & \text{in } \Omega, \end{cases} \tag{1}$$

where $g(s)$ is the decreasing smooth function, $g(0) = 1, 0 < g(s) \to 0$ for $s \to \infty$,

$$G_\sigma \in C^\infty\left(\mathbb{R}^2\right) \text{ is the smoothing kernel with } \int_{\mathbb{R}^2} G_\sigma(x)dx = 1.$$

Furthermore, $\lim_{\sigma\to 0} G_\sigma = \delta_x$, where δ_x is the Dirac measure at point x, $u_0 \in L^2(\Omega)$. Also

$$(G_\sigma * u)(x) = \int_{\mathbb{R}^2} G_\sigma(\mathrm{x} - \xi)\tilde{u}(\xi)d\xi,$$

where $\tilde{u}$ is an extension of u to $\mathbb{R}^2$ (see [1]) and $G_\sigma(x)$ is the Gaussian filter, given by

$$G_\sigma(x) = \frac{1}{2\pi\sigma^2}\exp\left(-\frac{\|x\|^2}{\sigma^2}\right), \quad x \in \mathbb{R}^2.$$

Moreover, in literature, different diffusion coefficients have been proposed. For example, Perona and Malik [5] provided some examples of g. They proposed diffusion functions of the type

$$g_1(s) = \frac{1}{1 + (s/\alpha)^2} \quad or \quad g_2(s) = e^{-(s/\alpha)^2}.$$

Note that in our case, we are interested in the first one with $\alpha = 1$.

In this paper, we utilize the finite volume method (referenced as [2]) to derive an approximation scheme for (1). This scheme is well-suited for image processing, as it employs discrete approximations that are piecewise constant, corresponding to control volumes aligned with the pixel structure of the image. Moreover, the scheme allows us to obtain L^2 a-priori estimates for fully discrete solutions. To establish convergence of the approximations towards the weak solution of (1), we rely on Kolmogorov's compactness theorem.

2 Literature Review

Several authors have already investigated the convergence of such studies. In 2001, K. Mikula and N. Ramarosy [4] examined the convergence of the semi-implicit finite volume scheme for the modified Perona-Malik nonlinear image selective smoothing equation (referred to as anisotropic diffusion in image processing) in the sense of Catt, Lions, Morel, and Coll. They established the convergence to the weak solution. Additionally, Z. Kriv [3] studied the explicit finite volume scheme and demonstrated its convergence to the weak solution of the continuous equation. However, this explicit scheme imposes certain restrictions on the scale step size, unlike the semi-implicit scheme.

3 Results and Discussion

We approximate the model as follows: Determine vectors $(u_K^n)_{K\in\mathcal{T}_h}$ and $(w_K^n)_{K\in\mathcal{T}_h}$ for $n \in \{0,\dots,N\}$, such that for all $K \in \mathcal{T}_h$ and $n \in \{0,\dots,N-1\}$

$$m(K)\frac{u_K^{n+1}-u_K^n}{\Delta t} - \sum_{L\in N(K)} g_{KL}^{\sigma,n}\left(\tilde{u}_h\right) T_{KL}\left(u_L^{n+1}-u_K^{n+1}\right) + 2\lambda m(K) w_K^{n+1} = 0, \tag{2}$$

$$m(K)\frac{w_K^{n+1}-w_K^n}{\Delta t} - \sum_{L\in N(K)} T_{KL}\left(w_L^{n+1}-w_K^{n+1}\right) = m(K)(u_K^{n+1}-u_K^0), \tag{3}$$

$$u_K^0 = \frac{1}{m(K)}\int_K u_0(x)dx, \quad g_{KL}^{\sigma,n}\left(\tilde{u}_h\right) := g\left(\left|\nabla G_\sigma * \tilde{u}_h\left(x_{KL},t_n\right)\right|\right), \tag{4}$$

where $\tilde{u}$ is a periodic extension of the discrete image calculated in the $n-$th scale step. For more details, see [3].

For the purpose of the analysis, we introduce "piecewise constant" functions.

$$u_h(x,t) = u_K^{n+1} \qquad \textit{and} \qquad w_h(x,t) = w_K^{n+1},$$

for all $(x,t) \in K \times (n\Delta t,(n+1)\Delta t]$, with $K \in \mathcal{T}_h$ and $n \in \{0,\dots,N-1\}$.

Our main result is

Theorem 3.1 *Assume that $u_0 \in L^2(\Omega)$. Then the finite volume solution (u_h,w_h), converges along a subsequence to (u,w) as $h \to 0$, where (u,w) is a weak solution of (1). The convergence is in the following sense:*

$$(u_h,w_h) \to (u,w) \text{ strongly in } L^2\left(\Omega_T\right) \text{ and a.e. in } \Omega_T,$$

$$\left(\nabla_h u_h, \nabla_h w_h\right) \to (\nabla u, \nabla w) \text{ weakly in } \left(L^2\left(\Omega_T\right)\right)^2.$$

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Controllability and Stabilization of Coupled Hyperbolic Systems

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Abstract:

In this talk, we present some new results on the boundary controllability and stabilization of systems of degenerate wave equations coupled by velocities. Our first main result asserts the controllability of the considered system for sufficiently large time T under the action of a boundary control acting on only one equation, if the coupling parameter is small enough. In the second main result, we prove the exponential decay of the total energy of the whole system in the case of indirect linear boundary damping, i.e. when only one equation is directly damped by a linear boundary feedback. The method of proof combines the multiplier method together with suitable Hardy-Poincaré inequalities.

Keywords: Controllability, Stabilization, Wave equations.

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On the existence of solution to nonlinear elliptic equation with variable exponent and singular lower order term

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Abstract:
In this paper, we prove the existence of weak solutions for a some nonlinear elliptic equations which contain a singular lower order gradient term and an $L^1(\Omega)$ datum, in the framework of Sobolev spaces with variable exponents. Additionally, we prove that the presence of the lower order term leads to a regularizing effect on the solutions,.

Keywords: Variable Sobolev spaces; Elliptic equation, Singular gradient lower order term.

1 Introduction

Let Ω bounded open subset of $\mathbb{R}^N$, $N > 2$, with Lipschitz boundary $\partial\Omega$, and $p, q \in C(\overline{\Omega})$, such that $(p^+, q^+) = (\max_{x\in\overline{\Omega}} p(x), \max_{x\in\overline{\Omega}} q(x))$, $(p^-, q^-) = (\min_{x\in\overline{\Omega}} p(x), \min_{x\in\overline{\Omega}} q(x))$. This paper focuses on studying the following nonlinear elliptic problem

$$\begin{cases} -\operatorname{div}\left(a(x)|\nabla u|^{p(x)-2}\nabla u\right) + b(x)u^{r(x)} = B\dfrac{|\nabla u|^{q(x)}}{u^\theta} + f & \text{in } \Omega, \\ u > 0 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{1}$$

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with the function f is a positive function belongin to $L^1(\Omega)$ and the measurable functions a, b satisfying for some positive numbers $\alpha, \beta, \mu,\ \nu$ the following conditions

$$0 < \alpha \leq a(x) \leq \beta,\ 0 < \mu \leq b(x) \leq \nu, \tag{2}$$

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Hirarchical control problem for the heat equation with dynamic boundary conditions

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Abstract:

This paper focuses on hierarchical control for the heat equation with dynamic boundary conditions with the main goal is letting the state near from a prescribed target in a fixed observation region while the null controllability is serving as the secondary objective. In other words, we reverse the roles of the leader and the follower that were taken into account in the recent article [**?**]. For this purpose, we combine some appropriate Carleman estimates and the Stackelberg strategy.

Keywords: Hirarchical control, Dynamic boundary conditions, Pareto strategy, Carleman estimates.

1 Introduction

In classical mono objective optimal control problem, one typically tackles a system modeled by an evolution equation, and aims to find a control that steers the initial state to a fixed target. Usually, we deal with systems with one control and the resolution process leads to minimizing a cost function that generally satisfies some reasonable and appropriate conditions. However, in a multi objective control problem, as its name implies, there is more than one objective. This means that more than one control is involved and several cost functions to be simultaneously optimized.

To solve a such problem, we use the notion of strategies. In the literature, the Pareto Strategy is used for non-cooperative games, Nash Strategy for cooperative ones, and the Stackelberg Strategy when the problem is hierarchical.

2 Literature Review

In the last decades, these strategies are intensively applied in the context of evolution equations control. In [?] and [?], J. L. Lions dealt with the Stackelberg strategy for the hyperbolic and parabolic equations. Afterwards, the authors in [?] and [?], combined the Nash and the Stackelberg strategies with approximate controllability as the main objective. From a theoretical and numerical point of view, the papers , [?] and [?] have addressed the above questions in the case of parabolic equations and the Burgers equation, respectively. Finally, let us mention that the Stackelberg-Nash strategy for Stokes systems has been studied in [?].

3 Stackelberg control problem

In this section, we show the existence and uniqueness of the follower and the leader and establish their characterization using an optimality system.

3.1 Existence and characterization of the follower

The system under consideration is

$$\begin{cases} \partial_t y - \Delta y + a(x,t)y = f1_{\mathcal{O}} + v1_{\omega} & \text{in } \Omega_T, \\ \partial_t y_{\Gamma} - \Delta y_{\Gamma} y_{\Gamma} + \partial_\nu y + b(x,t)y_{\Gamma} = 0 & \text{on } \Gamma_T, \\ (y(0), y_{\Gamma}(0) = (y_0, y_{\Gamma,0}), & \text{in } \Omega \times \Gamma. \end{cases} \tag{1}$$

We use the notations:

$$L\varphi = \partial_t \varphi - d\Delta\varphi, \quad L^*\varphi = -\partial_t\varphi - d\Delta\varphi$$
$$L_\Gamma \varphi_\Gamma = \partial_t \varphi_\Gamma - \delta\Delta_\Gamma \varphi_\Gamma, \quad L_\Gamma^* \varphi_\Gamma = -\partial_t \varphi_\Gamma - \delta\Delta_\Gamma\varphi_\Gamma$$

For $\Phi = (\varphi, \varphi_\Gamma), \Psi = (\psi, \psi_\Gamma) \in \mathbb{E}_1$, define the following bilinear and linear forms

$$B(\Phi,\Psi) = \int_{\Omega_T} \rho^{-2} L^*\varphi L^*\psi dxdt + \int_{\Gamma_T} \rho^{-2}\left(L_\Gamma^*\varphi_\Gamma + \partial_\nu\varphi\right)\left(L_\Gamma^*\psi_\Gamma + \partial_\nu\psi\right) d\sigma dt +$$
$$\int_{\mathcal{O}\times(0,T)} \rho_0^{-2}\varphi\psi\, dx\, dt,$$
$$\ell_v(\Phi) = \int_{\omega_T} v\varphi dxdt + \langle \Phi(0), Y_0\rangle_{\mathbb{L}^2}, \text{ with } Y_0 \in \mathbb{L}^2 \text{ and } v \in \mathcal{V}_\rho \text{ are fixed.}$$

We are now in position to announce a the main result of this part. function K.

Theorem 3.1. *For a fixed leader $v \in \mathcal{V}_\rho$, we have the following results.*

(i) There exists a unique follower $f^\star[v] \in \mathcal{F}_\rho$, minimum of K.

*(ii) Let $f^\star[v]$ be the minimum of K and $Y(\cdot, v, f^\star[v]) = (y(\cdot, v, f^\star[v]), y_\Gamma(\cdot, v, f^\star[v]))$ the solution to (**??**) with $f = f^\star[v]$, then one has*

$$f^\star[v] = -\,\rho_0^{-2} p\big|_{\mathcal{O}\times(0,T)}, \quad y = \rho^{-2} L^* p, \quad y_\Gamma = \rho^{-2}(L_\Gamma^\star p_\Gamma + \partial_\nu p) \tag{2}$$

where $P = (p, p_\Gamma)$ is the unique solution to the variational problem

$$\begin{cases} B(P, Q) = \ell_v(Q) \\ \forall Q \in \mathbb{E}_1, \quad P \in \mathbb{E}_1. \end{cases} \tag{3}$$

3.2 Existence and characterization of the leader

This section is devoted to show the existence and uniqueness of the leader. We stress that the existence and uniqueness of the follower give arise to the reaction map $v \mapsto F(v) = f^\star[v]$. In this way the functional J can be written as: $J(v, f) = J(v, F(v)) := G(v)$. It is not difficult to see that the real-valued function $v \mapsto G(v)$ is continuous, strictly convex and coercive on $\mathcal{V}_{\rho_0}$. So we have the following existence and uniqueness results.

Proposition 1. *There is a unique leader control $v^\star \in \mathcal{V}_{\rho_0}$ such that*

$$G(v^\star) = \inf_{v \in \mathcal{V}_{\rho_0}} G(v).$$

Now, we shall give a characterization result for the leader. To this end, let $v^\star$ be the leader and $f^\star[v^\star]$ the follower. Consider the following system:

$$\begin{cases} \partial_t y^\star - \Delta y^\star + a(x,t) y^\star = f^\star[v^\star] 1_{\mathcal{O}} + v^\star 1_\omega & \text{in } \Omega_T, \\ \partial_t y_\Gamma^\star - \Delta_\Gamma y_\Gamma^\star + \partial_\nu y^\star + b(x,t) y_\Gamma^\star = 0 & \text{on } \Gamma_T, \\ -\partial_t \phi^\star - \Delta \phi^\star + a(x,t)\phi^\star = \alpha\,(y^\star - y_d)\, 1_{\omega_d} & \text{on } \Gamma_T, \\ -\partial_t \phi_\Gamma^\star - \Delta \phi_\Gamma^\star + \partial_\nu \phi^\star + b(x,t)\phi_\Gamma^\star = 0 & \text{on } \Gamma_T, \\ (y^\star(0), y_\Gamma^\star(0)) = (\phi^\star(T), \phi_\Gamma^\star(T) = (0,0) & \text{in } \Omega \times \Gamma. \end{cases} \tag{4}$$

and the following variational problems:

$$\begin{cases} P^\star \in \mathbb{E}_1, \text{ S.t.} \\ B(P^\star, Q) = \displaystyle\int_{\omega\times(0,T)} v^\star q \, dx\, dt + \langle Y_0, Q(0)\rangle_{\mathbb{L}^2} = \ell_{v^\star}(Q) \quad \forall Q \in \mathbb{E}_1. \end{cases} \tag{5}$$

$$\begin{cases} \Psi^\star \in \mathbb{E}_1, \text{ S.t.} \\ B(\Psi^\star, Q) = -\displaystyle\int_{\mathcal{O}\times(0,T)} \rho_0^{-2} \phi^\star q \, dx\, dt \quad \forall Q \in \mathbb{E}_1. \end{cases} \tag{6}$$

These problems are well-posed and can be used to characterize the leader $v^\star$, more precisely, one has.

Theorem 3.2. *Let $v^\star$ be the leader given by Proposition **??**, and $f^\star[v^\star]$ the associated follower. Let $Y^\star = (y^\star, y_\Gamma^\star), P^\star = (p^\star, p_\Gamma^\star), \Phi^\star = (\phi^\star, \phi_\Gamma^\star)$ and $\Psi^\star = (\psi^\star, \psi_\Gamma^\star)$ be the solutions to the problems (**??**)-(**??**). Then we have*

$$\begin{cases} f^\star[v^\star] = -\ \rho_0^{-2} p^\star\big|_{\mathcal{O}\times(0,T)}, \\ y^\star = \rho^{-2} L^* p^\star, \quad y_\Gamma^\star = \rho^{-2}(L_\Gamma^* p_\Gamma^\star + \partial_\nu p^\star), \\ v^\star = (\rho_0^{-2}\phi^\star + \rho_0^{-2}\psi^\star)|_\omega. \end{cases}$$

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Dynamics of two-steps reversible enzymatic reaction under the new generalized Hattaf fractional derivative

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Abstract:
In this work, we present a chemical kinetics model based on dynamics of enzyme processes to better understand the catalytic action of enzymes in chemical reaction. To incorporate the memory effect, we apply the new generalized Hattaf fractional (GHF) derivative. The existence and uniqueness of solutions are obtained by fixed point theory. Finally, numerical simulations are given to support the analytical results.

Keywords: Enzymatic reaction, Hattaf-type fractional derivative, fixed point theory, numerical simulation.

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Parameter Estimation and Model Validation for a Mathematical Temporal Model of Tumour Growth

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Abstract:

Gastrointestinal stromal tumours (GISTs) are among the most common cancers, following breast cancer in women and prostate cancer in men. The ability of GISTs to bypass and resist treatment is a major challenge in cancer therapy. In this study, we present a temporal model based on ordinary differential equations to capture the temporal evolution of GISTs and their resistance to treatment. Our model is adapted to individual patients through clinical data and simulations. We demonstrate the potential of mathematical modelling in describing such phenomena. Additionally, we investigate whether the model can predict the onset of resistance to treatment and discuss the importance of treatment dosage in tumour development. Our findings may contribute to the development of personalized treatment strategies for GIST patients.

Keywords: Mathematical modelling, Tumours, Inverse problem, Vascularisation, ODE, Targeted therapy, GIST.

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Mathematical Modeling and Optimal Control of Chemotherapy Resistance in Cancer Treatment

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Abstract:
Chemotherapy resistance poses a significant challenge in cancer treatment, and mathematical models offer valuable insights into the dynamics of tumor growth in the presence of resistance. In this study, we propose a mathematical model comprising four interconnected ordinary differential equations that capture the growth of sensitive and resistant cells, vascularization, and drug concentration during chemotherapy. Additionally, the paper explores the model's equilibria, conducts stability analysis, and presents numerical simulations to investigate the influence of various parameters on tumor growth and immune response. Utilizing optimal control theory, the study designs personalized cancer treatment protocols to optimize treatment outcomes while minimizing side effects and toxicity.
The research underscores the potential of mathematical modeling and optimal control theory in enhancing cancer treatment strategies and delivers a comprehensive analysis of tumor-drug interactions. Our findings emphasize the significant impact of vascularization on tumor growth and the crucial roles of proliferation and mortality parameters in chemotherapy resistance. Overall, our model establishes a robust framework for studying tumor growth dynamics under chemotherapy resistance, contributing to the development of improved cancer treatment strategies.

Keywords: Tumor modeling, Chemotherapy resistance, Vascularization, stability, Numerical simulations.

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Spatiotemporal Dynamics of RNA Viruses in Presence of Immunity and Treatment: A Case Study of SARS-CoV-2

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Abstract:
In this work, we develop a mathematical model governed by partial differential equations (PDEs) to describe the spatiotemporal dynamics of RNA viruses like SARS-CoV2 that causes COVID-19. The well-posedness of the developed model is proved by establishing the global existence, uniqueness, nonnegativity and boundedness of solutions. Furthermore, the threshold parameters and the global asymptotic stability of equilibria are rigourously established.

Keywords: PDEs, RNA viruses, SARS-CoV-2, immunity, global stability.

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Leray–Schauder degree theory for anti-periodic Ψ-Caputo-type fractional p-Laplacian problems

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Abstract:
The main crux of this work is to study the existence of solutions for a certain type of nonlinear Ψ-Caputo fractional differential equations with anti-periodic boundary conditions and p-Laplacian operator. The proofs are based on the Leray–Schauder degree theory and some basic concepts of Ψ-Caputo fractional calculus. As an application, our theoretical result has been illustrated by providing a suitable example.

Keywords: Ψ-fractional integral, Ψ-Caputo fractional derivative, Leray-Schauder degree theory.

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Optimal Control of a Spatiotemporal Discrete Tuberculosis Model

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Abstract:
Understanding the impact of human behavior on the spread of infectious diseases might be the key to developing better control strategies. Tuberculosis (TB) is an infectious disease caused by bacteria that mostly affects the lungs. TB remains a global health issue due to its high mortality. The paper proposes a spatiotemporal discrete tuberculosis model, based on the assumption that individuals can be classified as susceptible, exposed, infected, and recovered (SEIR). The objective of this work is to introduce a strategy of control that will reduce the number of exposed and infected individuals. Three controls are established to accomplish this. The first control is a public awareness campaign that will educate the public on the signs, symptoms, and treatments of tuberculosis, allowing them to seek treatment if they are at risk. The second control initiates chemoprophylaxis efforts for people who are latently infected, and the third control characterizes the treatment effort for people who are actively infected. We have shown the existence of optimal controls to give a characterization of controls in terms of states and adjoint functions by using Pontryagin's Maximum Principle. Using numerical simulations, our results indicate that awareness campaigns should be combined with treatment and chemoprophylaxis techniques to reduce transmission. As a result, it demonstrates the efficacy of the suggested control strategies in reducing the impact of the disease.

Keywords: Discrete model, spatiotemporal, Tuberculosis, Optimal Control, Pontryagin's maximum principle, chemoprophylaxis.

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PDEs & Applications

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July 27-28, 2023, Faculty of Sciences Casablanca, Hassan II University, Casablanca Morocco

Existence results for φ-Caputo fractional differential equations with p-Laplacian operator via topological degree methods

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Abstract:
The primary objective of this paper is to examine the existence of solutions for anti-periodic nonlinear differential equations with φ-Caputo fractional derivative involving the p-Laplacian operator. The key findings of this study are established by employing topological degree methods, specifically condensing maps, along with various properties of φ-Caputo fractional calculus and measures of noncompactness. Furthermore, to demonstrate the practical relevance of our theoretical results, we present a nontrivial example at the conclusion of the paper.

Keywords: φ-fractional integral, φ-Caputo fractional derivative, topological degree methods.

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Interface Homogenization of a Periodic Array of Linear Viscoelastic Inclusions: A Matched Asymptotic Expansion Approach for Helmholtz problem

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Abstract:
This work examines the effective behavior of a periodic network composed of linear viscoelastic inclusions enclosed in a linear viscoelastic matrix. We provide a homogenization method that uses a matched asymptotic expansion technique to examine this complex system. We focus on the wave equation in the harmonic regime in relation to the shear wave question. We deduce the actual behavior of the system using the homogenization approach, focusing on displacement and normal stress at the interface. According to our study, the actual behavior is similar to that of an equivalent interface linked to jump conditions. We explicitly consider the scenario of rectangular inclusions to validate our findings. Transmission coefficients and displacement fields are successfully calculated in closed shapes. Moreover, by comparing our results with direct numerical simulations, we carefully assess the accuracy of our conclusions.

Overall, this study provides a better understanding of the behavior of periodic networks with linear viscoelastic inclusions. The homogenization method and corresponding asymptotic expansion technique used here provide a solid framework for describing the practical behavior of these complex systems, paving the way for further understanding and numerous research projects in this field and its applications.

Keywords: Interface Homogenization, Matched asymptotic expansion, effective jump conditions, Wave equation, Harmonic regime,Helmholtz problem, Shear waves, Effective behavior, Interface, Linear viscoelasticity, stress, Displacement.

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A viability result for non-convex differential inclusion in Banach spaces

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Abstract:

This paper deals with the existence of solutions to the following differential inclusion :

$$\begin{cases} \dot{x}(t) \in F(t, x(t)) \quad a.e.\ on\ [0, T[; \\ x(0) = x_0; \\ x(t) \in K \quad for\ \ all\ \ t \in [0, T], \end{cases}$$

where $F : [0, T] \times K \to 2^E$ is a non-convex multifunction and K is a closed subset of a separable Banach space E.

Keywords: viability, measurable multifunction, selection.

1 Introduction

Let E be a separable Banach space, K a nonempty closed subset of E, T a strictly positive real and put $I := [0, T]$. Let $F : I \times K \to 2^E$ be a multifunction measurable with respect to the first argument and uniformly continuous with respect to the second argument.

The aim of this work is to establish, for any fixed $x_0 \in K$, the existence of an absolutely continuous function $x(.) : I \to K$ satisfying :

$$\begin{cases} \dot{x}(t) \in F(t, x(t)) \quad a.e.\ on\ [0, T[; \\ x(0) = x_0; \\ x(t) \in K \quad for\ \ all\ \ t \in I. \end{cases} \tag{1.1}$$

Our approach is based on Euler's method, it consists of constructing a sequence of approximate solutions by using Lebesgue's Differentiation Theorem and selection's techniques.

2 Main result

Let $F : I \times K \to 2^E$ be a multifunction with nonempty closed values in E.

On F we make the following hypotheses :

(H_1) For each $x \in K$, $t \to F(t,x)$ is measurable.

(H_2) For all $t \in I$, $x \to F(t,x)$ is uniformly continuous as follows :

$$\forall\, \varepsilon > 0,\ \exists\, \delta(\varepsilon) > 0,\ \forall (t,x,y) \in I \times K \times K,$$
$$\|x - y\| \leq \delta(\varepsilon) \Rightarrow d_H(F(t,x), F(t,y)) \leq \varepsilon.$$

(H_3) There exists $M > 0$, for all $(t,x) \in I \times K$,

$$\|F(t,x)\| := \sup_{z \in F(t,x)} \|z\| \leq M.$$

(H_4) For all $t \in I$ and $x \in K$, for all measurable selection $\sigma(.)$ of the multifunction $t \to F(t,x)$

$$\liminf_{h \to 0^+} \frac{1}{h} d_K\left(x + \int_t^{t+h} \sigma(s)ds\right) = 0,$$

which is equivalent to

$$\liminf_{h \to 0^+} \frac{1}{h} e\left(x + \int_t^{t+h} F(s,x)ds, K\right) = 0.$$

where $e(.,.)$ denotes the Hausdorff's excess and $\displaystyle\int_t^{t+h} F(s,x)ds$ stands for the Aumann's integral of the multifunction $t \to F(t,x)$.

Let $x_0 \in K$. Under hypotheses (H_1)-(H_4) we shall prove the following result:

Theorem 2.1. *There exists an absolutely continuous function $x(.) : I \to E$, such that*

$$\begin{cases} \dot{x}(t) \in F(t, x(t)) \quad a.e. \text{ on } [0,T[; \\ x(0) = x_0; \\ x(t) \in K, \qquad \forall\, t \in I. \end{cases}$$

International Conference on Recent Advances in Partial Differential Equations and Applications (ICRAPDEA'23)

July 27-28, 2023, Faculty of Sciences Casablanca, Hassan II University, Casablanca Morocco

On the weak and strong stabilization of infinite dimensional bilinear systems with bounded control in Hilbert space

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Abstract:

This paper is concerned with the stabilization of infinite dimensional bilinear system $\partial_t y(t) = Ay(t) + u(t)By(t)$ where the linear operator A generates a strongly continuous semigroup of contractions $(S(t))_{t\geq 0}$ on a real Hilbert space H and $B : H \to H$ is a linear bounded operator. Then, we give sufficient conditions for weak and strong stabilization of such a system using bounded control $u \in [0, 1]$. Applications to partial differential equations are presented.

Keywords: Infinite dimensional systems, stabilization, feedback control, semigroups theory.

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International Conference on Recent Advances in Partial Differential Equations and Applications (ICRAPDEA'23)

July 27-28, 2023, Faculty of Sciences Casablanca, Hassan II University, Casablanca Morocco

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DEGENERATE ELLIPTIC NONLINEAR PROBLEM WITH A SINGULAR SOURCE TERM

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Abstract:
In this paper, we prove existence and regularity results for solutions of some nonlinear Dirichlet problems for an elliptic equation with source term having a singularity at the origin.

$$\begin{cases} -\operatorname{div}(a(x,u,\nabla u)) & = \mu h(u) \quad \text{in } \Omega \\ u > 0 & \quad\quad\quad\quad\ \text{in } \Omega \\ u = 0 & \quad\quad\quad\quad\ \text{on } \delta\Omega \end{cases} \tag{1}$$

where Ω is bounded open subset of $\mathbb{R}^N (N \geq 2)$, $\mu \in W^{-1,p'}(\Omega)$ and the singular sourcing $h : (0,\infty) \longrightarrow (0,\infty)$ is continuous and bounded.

Keywords: Degenerate elliptic equation, singular nonlinearity.

1 Introduction

The aim of this work is the study the boundary value problem as given in (1). Let us give the precise assumptions on the problems that we will study. Ω is bounded open subset of $\mathbb{R}^N (N \geq 2)$, μ is a nonnegative function that belongs to $W^{-1,p'}(\Omega)$ and the singular sourcing $h : (0,\infty) \longrightarrow (0,\infty)$ is continuous and bounded. Let $a(x,t,\xi) : \Omega \times \mathbb{R} \times \mathbb{R}^N \to \mathbb{R}^N$ be a Carathéodory function satisfying for $\nu \in L^{p'}(\Omega), \nu \geq 0, \alpha, \beta > 0$,

$$a(x,t,\xi) \cdot \xi \geq \alpha |\xi|^p \text{ a.e. } x \text{ in } \Omega, \in \mathbb{R}, \forall \xi \in \mathbb{R}^N, \tag{2}$$

$$a(x,t,\xi) - a(x,t,\xi') \cdot (\xi - \xi') \geq 0 \ \text{ a.e. } x \text{ in } \Omega,\ \forall t \in \mathbb{R} \forall \xi,\ \xi' \in \mathbb{R}^N, \tag{3}$$

$$|a(x,t,\xi))| \leq \beta(\nu(x) + |t|^{p-1} + |\xi|^{p-1}) \text{ a.e. } x \text{ in } \Omega, \forall t \in \mathbb{R}, \forall \xi \in \mathbb{R}^N. \tag{4}$$

We will rewrite our problem in this form

$$\begin{cases} u \in W_0^{1,p}(\Omega) \ , \\ \langle Au,\ v\rangle = \langle \mu,\ h(u)v\rangle \ \forall v \in V, \end{cases} \tag{5}$$

where $\mu \in W^{-1,p'}(\Omega)$ and V is some space allowing the equality above to hold in the distributional sense and

$$\langle Au,\ v\rangle = \int_\Omega a(x,u,\nabla u)\cdot\nabla v dx. \tag{6}$$

2 Literature Review

These types of problems have been examined in a linear case.– i.e. when the operator A is linear in [1], whose model is

$$\begin{cases} u \in W_0^{1,2}(\Omega)\ , \\ \int_\Omega A(x)\nabla u\cdot\nabla v dx = \langle \mu,\ h(u)v\rangle\ \forall v \in V, \end{cases} \tag{7}$$

where V is a space of functions containing the Schwartz-space $\mathcal{D}(\Omega)$ of C^∞-functions with compact support in Ω.
The nonlinear case has treated in [2] whose form is

$$\begin{cases} u \in W_0^{1,p}(\Omega)\ , \\ \langle Au,\ v\rangle = \langle \mu,\ h(u)v\rangle\ \forall v \in V, \end{cases} \tag{8}$$

where $\mu \in W^{-1,p'}(\Omega)$ and V is some space allowing the equality above to hold in the distributional sense, and for $u, v \in W_0^{1,p}$

$$\langle Au,\ v\rangle = \int_\Omega a(x,\nabla u)\cdot\nabla v dx. \tag{9}$$

3 Results and Discussion

Theorem 3.1 *Let $\mu \in W^{-1,p'}(\Omega)$ be a nonnegative bounded measure, A be an operator satisfying (2), (3) and (4). Let h be unbounded near 0.*
h: $\mathbb{R}^+ \to \mathbb{R}^+$ a nonnegative, nonincreasing function such that

$$\begin{cases} \lim_{s\to 0^+} h(s) = +\infty, \\ \forall\epsilon > 0,\ h \textit{ is Lipschitz continuous on } (\epsilon,\ +\infty)\ , \end{cases} \tag{10}$$

$$\exists\mathcal{K} : \mathbb{R}^+ \to \mathbb{R}^+ \textit{ s.t. } h \le \mathcal{K} \textit{ and s.t. } (s\mathcal{K}(s)) \in W^{1,\infty}(\mathbb{R}^+). \tag{11}$$

Then there exists u solution to

$$\begin{cases} u \in W_0^{1,p}(\Omega)\ ,\ u \ge 0, \\ \langle Au,\ v\rangle = \langle \mu,\ h(u)v\rangle\ \forall v \in C_c^1(\Omega)\ . \end{cases} \tag{12}$$

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International Conference on Recent Advances in Partial Differential Equations and Applications (ICRAPDEA'23)
July 27-28, 2023, Faculty of Sciences Casablanca, Hassan II University, Casablanca Morocco

Entropy solution for nonlinear parabolic problems with growth condition and two lower order terms in Musielak spaces

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Abstract: In this current article, we investigate the existence of solutions for specific nonlinear parabolic models with L^1 data inside the context of Musielak spaces, where the operator a acts from $W_0^{1,x}L_\varphi(Q)$ to its dual, and two lower order terms, φ is a Musielak function. The first component $g(x,t,u,\nabla u)$ is supposed to satisfy a sign condition on u and a growth condition on ∇u in the first term, whereas the second term $H(x,t,\nabla u)$ is only growing at most as $\overline{\gamma}_x^{-1}\gamma_x(|\nabla u|)$; γ is a Musielak function. Keep in the mind that γ does not assume the Δ_2 condition and the source term f belongs to $L^1(Q)$.

Keywords: Lower order term, Musielak-Sobolev spaces, Parabolic equations, Natural growth.

1 Introduction.

Let Ω be a bounded Lipschitz domain of $\mathbb{R}^N$ $(N \geq 2)$ and $Q = \Omega \times (0,T), T > 0$. We treate the nonlinear parabolic problem given by:

$$\begin{cases} \frac{\partial b(u)}{\partial t} + A(u) + g(x,t,u,\nabla u) + H(x,t,\nabla u) = f, & \text{in } Q \\ u = 0 & \text{on } \partial\Omega \times (0,T) \\ b(u)(t=0) = b(u_0) & \text{in } \Omega, \end{cases} \tag{1}$$

where

$$b : \mathbb{R} \longrightarrow \mathbb{R} \text{ is an increasing } \mathcal{C}^1 \text{ function satisfying}$$

$$b(0) = 0 \text{ and } 0 < b_0 \leq b'(s) \quad \forall s \in \mathbb{R},$$

where b_0 is given real number.

The mapping $A : D(A) \subset W_0^{1,x}L_\varphi(\mathrm{Q}) \to W^{-1,x}L_{\bar{\varphi}}(\mathrm{Q})$ is a differential operator given by

$$A(u) = -\text{ div } a(x,t,u,\nabla u)$$

with $a : \Omega \times [0,T] \times \mathbb{R} \times \mathbb{R}^N \longrightarrow \mathbb{R}^N$ and φ is a Caratheodory function. φ is the conjugate Musielak function of φ.

The nonlinear term $g(x,t,u,\nabla u)$ satisfies the growth condition

$$|g(x,t,s,\xi)| \leq q(|s|)\,(c_2(x,t) + \varphi(x,|\xi|));$$

and the sign condition

$$g(x,t,s,\xi)\,s \geq 0;$$

where q is a positive, continuous and non-decreasing function, while c_2 is a Lebesgue function.
Moreover, we assume that the Caratheodory function H satisfies:

$$|H(x,t,\xi)| \leq P(x,t)\,\overline{\gamma}_x^{-1}\gamma_x(|\xi|),$$

for all $P \in L^\infty(Q)$, where γ is a Musielak function. The source term f is assumed to be of $L^1(Q)$ regularity.

Numerous studies have addressed this problem, and while it is not feasible to mention all them, we will choose and provide the results essential to our work:

2 Literature Review

The parabolic problem of this type in Lebesgue spaces was studied by Dall'aglio and Orsina [6] with g and H equal to 0, $b(u) = u$. On the other hand, the case H equal to 0 was studied by [7].

In the framework of variable exponent Sobolev spaces, Bendahmane et al [5] demonstrated the well-posedness by establishing the existence of a renormalized solution for the nonlinear parabolic problem (1) in the framework of Lebesgue Sobolev spaces, and by considering $b(u) = u$ and $H \equiv 0$. Moreover, in [4], Azroul et al proved the existence of a renormalized solution without the sign and coercivity conditions on g, with $b(u_0) \in L^1(\Omega)$, and $H \equiv 0$. In their contribution, they treated a class of doubly nonlinear parabolic equations with variable exponents.

For Musielak spaces, Benkirane et al affirmed the existence of entropy solutions to problem (1) for a general Musielak function φ, $b(u) = u$, and $H \equiv 0$. In [3], Ait Khellou et al analyzed problem (1) without assuming a sign condition on g. At last, In [2] Ait Khellou et al established the existence of solution to problem (1) with $H \equiv 0$ and $g(x,t,s,\xi) = -div\ \Phi(x,t,s)$ (Φ is a Caratheodory function) by assuming essentially the log-Hôlder continuity condition on the Musielak function.

3 Results and Discussion

We shall prove the following existence theorem

Theorem 1 *Under the assumptions of introduction, there exists at least one solution of the problem corresponding to* (1) *in the following sense*

$$\begin{cases} T_k(u) \in W_0^{1,x}L_\varphi(Q), \quad g(x,u,\nabla u) \in L^1(\Omega), \quad H(x,\nabla u) \in L^1(\Omega), \\ \displaystyle\int_\Omega S_k(b(u(\tau)) - v(\tau))\,dx + \int_0^\tau \langle \frac{\partial v}{\partial t}, T_k(b(u) - v)\rangle\,dt + \int_{Q_\tau} a(x,t,u,\nabla u)\,\nabla T_k(b(u) - v)\,dxdt \\ \displaystyle + \int_{Q_\tau} (g(x,t,u,\nabla u) + H(x,t,\nabla u))\,T_k(b(u) - v)\,dxdt \\ \displaystyle \leq \int_\Omega f\,T_k(b(u) - v)\,dxdt + \int_\Omega S_k(b(u_0) - v(0))\,dx, \\ \forall v \in W_0^{1,x}L_\varphi(Q) \cap L^\infty(Q),\ \textit{such that } \frac{\partial v}{\partial t} \in W^{-1,x}L_{\bar{\varphi}}(Q) + L^1(Q)\ \forall k > 0, \tau \in (0,T). \end{cases} \tag{2}$$

Where $Q_\tau = \Omega \times (0,\tau)$, $T_k(s) = \max(-k, \min(k, s))$. *and* $S_k(r) = \displaystyle\int_0^r T_k(s)\, ds$, *for all* $s \in \mathbb{R}$ *and* $k \geq 0$.

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International Conference on Recent Advances in Partial Differential Equations and Applications (ICRAPDEA'23)

July 27-28, 2023, Faculty of Sciences Casablanca, Hassan II University, Casablanca Morocco

REMEDIABILITY PROBLEM IN FINITE DIMENSION LINEAR FRACTIONAL DYNAMICAL SYSTEMS

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Abstract:
We study with respect to the observation, the possibility of finite time compensation of known or unknown disturbances. Under convenient hypothesis, we show how to find the optimal control ensuring the compensation of a disturbance, by bringing back the corresponding observation to the normal one. This concept is also examined as minimization problem with a decent cost function. A comparison between the two approaches is given and the relation with the notions of controllability is studied. Various situations are also examined.

Keywords: Fractional linear disturbed systems, Dynamical systems, remediability, observation, optimal control, disturbance.

1 Introduction

Perturbations can generate significant damage to the dynamic system (infections, pollution, etc.) in various domain. Disturbed systems have continued to grow in importance in recent years. Unknown disturbances are detected by observation and several works have been devoted to their detection and reconstruction from the corresponding observation. (see [2], [1], [4]).
However, the detection of a disturbance is generally insufficient, it is however necessary to act by means of commands to regulate or attenuate its impact on the system. The notion of remediability consists in studying the existence of an adequate control ensuring the remediability or the compensation of possible disturbances by attenuating it, and this by bringing back

the observation of the disturbed system towards its state without disturbance.
The notions of remediability and effective actuators are studied and treated firstly for a class of parabolic systems in the case of a finite time horizon, for discrete systems, hyperbolic systems, regional and asymptotic cases, we can see [5], [6]. In this works, the authors study, in relation to observation, the existence of a control ensuring the compensation of any disturbance. They also proved that the remediability is weaker and more flexible than the controllability of the system. In [3] the authors was defined the gradient remediability of distributed parabolic systems and the relationship with the gradient controllability.

In this work, we consider a class of finite dimension time invariant fractional order control systems described by a linear state equation as follows :

$$\begin{cases} {}_0^cD_t^\alpha z(t) = Az(t) + Bu(t) + f(t) \ ; \ 0 < t < T \ ; \ 0 < \alpha \leq 1 \\ z(0) = z_0 \end{cases} \tag{1}$$

where $A \in M_n(\mathbb{R})$, $B \in M_{n,p}(\mathbb{R})$, $u \in L^2(0,T;\mathbb{R}^p)$, $f \in L^2(0,T;\mathbb{R}^n)$ and ${}_0^cD_t^\alpha$ denotes the Caputo fractional order derivative, where

$${}_0^cD_t^\alpha z(t) = \begin{cases} \frac{1}{\Gamma(1-\alpha)} \displaystyle\int_0^t (t-s)^{-\alpha} \dot{z}(s) ds, & 0 < \alpha \leq 1 \\ \dot{z}(t) & \alpha = 1 \end{cases}$$

The system (1) is augmented by the output equation:

$$y(t) = Cz(t) \ ; \ \ 0 < t < T \tag{2}$$

with $C \in M_{q,n}(\mathbb{R})$, we have

$$z(t) = \Phi_0(t) z_0 +_\alpha H_t u +_\alpha G_t f$$

where

$$\Phi_0(t) = \sum_{k=0}^{\infty} \frac{A^k t^{k\alpha}}{\Gamma(k\alpha + 1)}$$

and Γ is Gamma function. Then

$$y(t) = C\Phi_0(t) z_0 + C_\alpha H_t u + C_\alpha G_t f$$

where ${}_\alpha H_t$ and ${}_\alpha G_t$ are the operators defined by

$$\begin{aligned} {}_\alpha H_t : \quad L^2(0,t;\mathbb{R}^p) \quad &\longrightarrow \quad \mathbb{R}^n \\ u \qquad &\longrightarrow \quad \int_0^t \Phi(t-s) B u(s) ds \end{aligned} \tag{3}$$

and

$$
\begin{array}{rccc}
{}_{\alpha}G_t : & L^2(0,t;\mathbb{R}^n) & \longrightarrow & \mathbb{R}^n \\
 & f & \longrightarrow & \displaystyle\int_0^t \Phi(t-s)f(s)ds
\end{array}
\tag{4}
$$

with

$$
\Phi(t) = \sum_{k=0}^{\infty} \frac{A^k t^{(k+1)\alpha-1}}{\Gamma[(k+1)\alpha]}
$$

let us first recall the controllability and its rank characterization. The system

$$
\left\{
\begin{array}{l}
{}_0^cD_t^{\alpha} z(t) = Az(t) + Bu(t) \ ; \ 0 < t < T \ ; \ 0 < \alpha \leq 1 \\
z(0) = z_0
\end{array}
\right.
\tag{5}
$$

is controllable on $[0,T]$, if for any initial state z_0 and desired state z_d in $\mathbb{R}^n$, there exists a control $u \in L^2(0,T;\mathbb{R}^p)$ such that:

$$
z(T) = z_d
$$

This is equivalent to

$$
Im({}_{\alpha}H_T) = \mathbb{R}^n
$$

also, the matrix

$$
{}_{\alpha}\mathfrak{C}(T) = \int_0^T \Phi(T-s)BB^*\Phi(T-s)^*(T-s)^{2(1-\alpha)}ds
$$

is invertible
or (using Cayley-Hamilton Theorem)

$$
rank \begin{pmatrix} B & AB & \dots & A^{n-1}B \end{pmatrix} = n.
$$

In the case without disturbance and control, i.e. $f = 0$ and $u = 0$ the observation is given by

$$
y_{0,0}(t) = C(t)\Phi_0(t)z_0.
$$

But if the system is disturbed by a term f, the observation becomes

$$
y_{0,f}(t) = C\Phi_0(t)z_0 + \int_0^t C\Phi(t-s)f(s)ds \neq C\Phi_0(t)z_0.
$$

Then we introduce a control term Bu in order to reduce the effect of this disturbance at final time T, i.e. $y_{u,f}(T) = y_{0,0}(T)$. The system (1) augmented with the output (2), or (1)+ (2) is said to be remediable on $[0, T]$, if for any $f \in L^2(0, T; \mathbb{R}^n)$, there exists a control $u \in L^2(0, T; \mathbb{R}^p)$ such that:

$$C_\alpha H_T u + C_\alpha G_T f = 0$$

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ON TWO POINT BOUNDARY-VALUE PROBLEMS FOR DIFFERENTIAL INCLUSIONS with $\phi-$Laplacian

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Abstract: We show the existence of solutions to boundary-value problems for differential inclusion $(\phi(x'(t)))' \in F(t, x(t))$, where $F(.,.)$ is a closed multi-function, measurable in t, Lipschitz continuous in x and ϕ is an homeomorphism function. We use the fixed point theorem introduced by Covitz and Nadler for contraction multivalued maps.

Keywords: Boundary value problems, contraction, measurability, multifunction.

1 Introduction

In this presentation, we study the existence of solutions of the following two boundary-value problem:

$$\begin{cases} (\phi(x'(t)))' \in F(t, x(t)) \text{ a.e. on } [0, T]; \\ x(0) = x'(\eta). \end{cases} \tag{1}$$

where $F : [0, T] \times \mathbb{R} \to 2^{\mathbb{R}}$ is a closed multivalued map, measurable with respect to the first argument and Lipschitz with respect to the second argument, and $\phi : \mathbb{R} \to \mathbb{R}$ is an homeomorphism function, $T > 0$, and $\eta \in]0, T[$.

2 Main results

We consider the following hypotheses:

(H1) $F : [0, T] \times \mathbb{R} \to 2^{\mathbb{R}}$ is a multi-valued map with nonempty closed values satisfying

(i) For each $x \in \mathbb{R}, t \mapsto F(t,x)$ is measurable;

(ii) There exists a function $m(.) \in L^1([0,T], \mathbb{R}^+)$ such that for all $t \in [0,T]$ and for all $x_1, x_2 \in \mathbb{R}$,

$$H(F(t,x_1), F(t,x_2)) \leq m(t)|x_1 - x_2|;$$

(H2) $\phi : \mathbb{R} \to \mathbb{R}$ is an homeomorphism, such that ϕ^{-1} is k-Lipschitz with $k > 0$;

(H3) For $\eta \in]0,T[$,

$$L(\eta) + TL(T) < \frac{1}{k},$$

where $L(t) = \int_0^t m(s)ds$ for all $t \in [0,T]$.

Theorem 2.1 *If assumptions (H1), (H2) and (H3) are satisfied, then the problem (1) has at least one solution x on $[0,T]$.*

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Finite-time stabilisation of a class of linear and bilinear control systems

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Abstract:
Based on the idea of Lyapunov function, we investigate the finite-time stability of some classes of abstract bounded bilinear systems using feed-back control laws. We first design a stabilising feedback control. Then, we study the well-posedness of the system in closed-loop by using the theory of semilinear evolutions equations. Then we proceed to the question of finite-time stability in a settling time of the system at hand. The obtained results are also applied to derive finite time stabilisation results for linear systems. Many examples are treated.

Keywords: Semilinear evolutions equations, linear systems, bi- linear systems, finite time stability.

International Conference on Recent Advances in Partial Differential Equations and Applications (ICRAPDEA'23)

July 27-28, 2023, Faculty of Sciences Casablanca, Hassan II University, Casablanca Morocco

How SIEM Technology and Big Data are Revolutionizing Cybersecurity with the Aid of Artificial Intelligence

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Abstract:
Cyber attacks have become more sophisticated and large scale over the years, making it difficult for traditional security measures to detect and respond to them effectively. This has led to a renewed interest in combining security information and event management (SIEM) technology with big data and AI to provide improved cyber defense. This article will explore how these technologies are being used to revolutionize the way organizations protect their networks and data from malicious activity. It will discuss the advantages of using SIEM technology with big data and IA, and how they can be leveraged together to form an effective cybersecurity perimeter. Additionally, it will look at the potential drawbacks of adopting these technologies, and the challenges that may need to be addressed in order for them to be effective.

Keywords: SIEM, IA, BIG DATA.

1 Introduction

In today's interconnected world, cybersecurity has become a critical component for organizations to safeguard their valuable data and infrastructure. With the increasing volume and complexity of cyber threats, traditional security approaches are no longer sufficient. This has led to the emergence of innovative technologies that are revolutionizing the field of cybersecurity, namely Security Information and Event Management (SIEM) and big data analytics.

SIEM systems combine real-time monitoring, threat intelligence, and incident response capabilities to provide comprehensive visibility into an organization's security posture. By collecting, correlating, and analyzing data from various sources such as network devices, servers, and applications, SIEM solutions enable security teams to detect and investigate potential security incidents rapidly. However, the sheer volume of data generated by modern IT environments often overwhelms traditional SIEM systems, making it challenging to identify and respond to threats effectively.

This is where big data analytics comes into play, offering the ability to process and analyze large and diverse datasets quickly. By leveraging advanced algorithms and machine learning techniques, big data analytics can identify patterns, anomalies, and hidden relationships within vast amounts of security data. This enables security professionals to detect and respond to threats more accurately and promptly, reducing the time it takes to detect and mitigate potential attacks.

The integration of artificial intelligence (AI) further enhances the power of SIEM and big data analytics in the cybersecurity landscape. AI-powered systems can automatically analyze vast amounts of security data, learn from past incidents, and adapt their defenses to emerging threats. This helps security teams stay ahead of attackers and respond proactively to potential vulnerabilities.

The combination of SIEM, big data analytics, and AI is transforming cybersecurity by providing organizations with proactive and intelligent threat detection capabilities. By leveraging the power of these technologies, organizations can detect and respond to threats in real-time, mitigating potential risks and minimizing the impact of cyberattacks.

This article will delve deeper into the revolutionizing impact of SIEM and big data analytics in the cybersecurity landscape. We will explore the key benefits and challenges associated with these technologies and discuss how they can work together to strengthen organizations cybersecurity posture. Additionally, we will examine real-world examples of organizations successfully implementing these technologies and the lessons learned from their experiences.

The integration of SIEM, big data analytics, and AI is shaping the future of cybersecurity. This article aims to shed light on the transformative capabilities of these technologies and provide insights into how organizations can leverage them to stay ahead in the ever-evolving threat landscape.

2 Overview of cybersecurity challenges and the need for advanced solutions

Cybersecurity challenges have increased significantly with the rapid growth of digital technology and the widespread use of the internet. Today, organizations face various threats like malware, ransomware, phishing attacks, data breaches, and advanced persistent threats (APTs). These challenges have a severe impact on businesses, governments, and individuals, leading to financial losses, reputational damage, and privacy breaches.

The need for advanced cybersecurity solutions arises from the evolving tactics used by cybercriminals. Traditional security measures like firewalls and antivirus software are no longer enough to protect against sophisticated attacks. Cybercriminals employ advanced techniques, exploit vulnerabilities in software and systems, and use social engineering tactics to deceive users.

One significant challenge is the increasing interconnectedness of devices in the Internet of Things (IoT) ecosystem. As more devices are connected to networks, there are more entry points for cyberattacks. Securing IoT devices is crucial to prevent unauthorized access and potential disruption of critical systems.

Additionally, the rise in remote work has created new challenges for cybersecurity. With employees accessing corporate networks from different devices and locations, there is an increased risk of unauthorized access and data leakage. Protecting remote workers' devices and ensuring secure communication channels are important aspects of cybersecurity.

Moreover, the increasing sophistication of hacking techniques and the emergence of nation-state-sponsored attacks increase the importance of advanced cybersecurity solutions. APTs target critical infrastructure, government institutions, and large organizations, aiming to gain unauthorized access, steal sensitive data, or disrupt operations[1].

The need for advanced solutions is also driven by the proliferation of cloud services and the requirement to protect data stored in the cloud. Cloud security solutions must ensure data privacy, encryption, and access control to safeguard sensitive information.

To address these challenges, advanced and multifaceted cybersecurity solutions are being developed. These solutions incorporate technologies like artificial intelligence (AI) and machine learning (ML) to detect and mitigate threats in real-time. AI and ML algorithms can analyze enormous amounts of data and identify patterns that indicate potential cyberattacks, enabling proactive threat prevention.

Advanced solutions also incorporate behavioral analytics, threat intelligence, and user behavior analysis to identify anomalous activities and potential insider threats within an organization. Implementing comprehensive security measures like network segregation, data encryption, secure coding practices, and regular security audits are essential components of an advanced cybersecurity strategy.

Overall, the increasing complexity and severity of cyber threats necessitate the adoption of advanced cybersecurity solutions to protect against evolving attacks and safeguard critical digital assets.

3 Results and Discussion

The combination of Siem, big data, and AI has the potential to revolutionize cybersecurity. By leveraging the power of data analytics, Siem can provide better threat detection, predictive analysis, and incident response capabilities. This helps organizations stay ahead of cyber threats and minimize the impact of potential attacks. Additionally, Siem facilitates the sharing of threat intelligence, promoting collaboration and collective defense against evolving cyber threats. As technology advances further, the integration of Siem and big data with AI will continue to play a crucial role in safeguarding digital assets and ensuring robust cybersecurity[2].

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International Conference on Recent Advances in Partial Differential Equations and Applications (ICRAPDEA'23)

July 27-28, 2023, Faculty of Sciences Casablanca, Hassan II University, Casablanca Morocco

Regional controllability of fractional-order dynamical system with time-varying delays in the state

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Abstract:
In this work, the regional controllability problem for fractional-order dynamical systems with time-varying delays in the state was considered using Caputo's standard derivative, in order to prove their regional exact and weak controllability. And also Depending on the Hilbert uniqueness method, we concentrate on the determination of control achieving regional controllability with minimum energy. In order to establish the credibility of our key findings, we examine two instances of fractional partial differential systems as illustrations-namely, the zonal actuator and the pointwise actuator.

Keywords: Regional controllability; Fractional-order systems; Caputo derivatives; Timevarying delays in the state; Optimal control; Minimum energy

1 Introduction & Motivation

Fractional Order Calculus (FOC) constitutes an branch of mathematics dealing with differentiation and integration under an arbitrary order of the operation, i.e. the order can be any real or even complex number.

Fractional-order derivatives are a generalization of classical integer-order ones.Mathematical modeling of systems and processes with the use of fractional-order derivatives leads to fractional differential equations.

Fractional differential equations occur in mathematical models of, among other things, mechanical, biological, chemical and medical phenomena. It has become apparent that fractional-order models reflect the behavior of many real-life processes more accurately than integer-order ones.

In many processes, future states depend on both the present state and past states of a system. This means that models describing the processes involve delays in state or in control.

Numerous mathematical models describe dynamical systems with delays in control, or both the state and control. Therefore, studying the properties of systems with delays is especially important.

Due to the large number of mathematical models describing time-delayed dynamical systems, the controllability of dynamical systems plays a crucial role in their analysis. In recent years, various controllability problems for different types of fractional order dynamical systems have been considered in many publications.

2 Conclusions and Results

We discussed the controllability of fractional systems with delay in the state, the solution expression for such systems is given for the regional controllability. Necessary and sufficient conditions for controllability of fractional systems with time-varying delays in the state are given. Control of minimum energy that assure the regional controllability is computed and given explicitly.

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International Conference on Recent Advances in Partial Differential Equations and Applications (ICRAPDEA'23)

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Boundary optimal control problem of semi-linear systems

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Abstract:
The present work deals with an optimal control problem of a semi-linear equation evolving in a spatial domain Ω. This equation is excited by bilinear time controls on the boundary $\partial\Omega$ of Ω. Therefore, We prove that an optimal control exists and is characterized as a solution of an optimality system. Furthermore, applications to different sets of controls are discussed. The approach used leads to an algorithm for computing such a control. Illustrations by simulations are also provided.

Keywords: Infinite dimensional non linear systems, boundary optimal control problem, bilinear optimal control.

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July 27-28, 2023, Faculty of Sciences Casablanca, Hassan II University, Casablanca Morocco

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Invariant sets for a class of semilinear delay differential equations with non-dense domain

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Abstract:
Let Z be a Banach space, $A : D(A) \subset Z \longrightarrow Z$ be a linear operator on Z and let q be real number such that $q > 0$. Denote by D a locally closed subset of Z and $F : [0, +\infty[\times \mathcal{C}\left([-q, 0], \overline{D(A)}\right) \longrightarrow Z$ a continuous function. A sufficient condition for D to be a forward invariant set of the semilinear differential equation of retarded type

$$\begin{cases} \frac{du(t)}{dt} = Au(t) + F\left(t, u_t\right), & \text{for} \quad t \geqslant 0, \\ u_0 = \varphi \in \mathcal{C}([-q, 0], Z_0), \end{cases}$$

is the tangency condition namely: there exists a bounded linear operator $B : \overline{D(A)} \longrightarrow Z$ such that for every $\xi > 0$ and $\tau > 0$ there exists $\gamma > 0$ such that that:

$$\lim_{h \to 0^+} \frac{1}{h} d\left(T_{(A-\gamma B)_0}(h)\varphi(0) + S_{A-\gamma B}(h)[F(t, \varphi) + \gamma B\varphi(0)], D\right) = 0,$$

for every $t \in [0, \tau]$ and every $\varphi \in \mathcal{C}([-q, 0], \overline{D(A)})$ with $\|\varphi\| \leqslant \xi$ and $\varphi(0) \in D$.

Keywords: Retarded differential equations, mild solutions, invariant sets.

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Discontinuous Galerkin approach method to solve the two-dimensional multilayer shallow water flows

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Abstract:
A Discontinuous Galerkin method is presented for the numerical solution of multilayer shallow water equations with mass exchange. The current model allows mass exchange between the layers, in contrast to typical models for multi-layered hydrostatic shallow water flows where immiscible suppression is assumed. The source terms in the multilayered shallow water equations comprise a set of conservation rules for which computing the eigenvalues is not simple. The governing equations are reformulated as a nonlinear system of conservation laws with differential source forces and reaction terms. The nodal polynomial basis functions of any order in space are used to discretize and solve the continuous equations locally in an unstructured computing environment. We select the numerical flux based on the regional Lax-Friedrichs flow, which creates connections between the elements, to finish the discretization in space. The numerical discretization results in a set of coupled nonlinear equations which can be solved efficiently and locally by integrating them using a Runge–Kutta scheme. The considered discontinuous Galerkin method is fully explicit, stable, highly accurate, and locally conservative finite element method whose approximate solutions are discontinuous across inter-element boundaries; this property renders the method ideally suitable for the hp- adaptivity. Several numerical results are presented to demonstrate the high resolution of the proposed method and to confirm its capability to provide accurate and efficient simulations to solve the two- dimensional multilayer shallow water equations.

Keywords: Multilayer shallow water equations, Discontinuous Galerkin method, Finite element method, Runge–Kutta scheme.

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International Conference on Recent Advances in Partial Differential Equations and Applications (ICRAPDEA'23)

July 27-28, 2023, Faculty of Sciences Casablanca, Hassan II University, Casablanca Morocco

On the exponential stabilization for a class of the second order semilinear distributed systems with time delay

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Abstract:
This paper deals with the problem of exponential stabilization for a class of distributed second order semilinear systems with time delay in Hilbert state space governed by a bounded feedback control. The existence and uniqueness of mild solution for the considered systems are proved. Sufficient and necessary conditions for exponential stabilization are given. Some illustrating applications are provided.

Keywords: Exponential stabilization, Second order, Distributed semilinear systems, Time delay.

1 Introduction

The importance of feedback stabilization for infinite dimensional bilinear controlled systems in presence of constant time delay explains the growing interest that researchers present to this subject (see e.g. [1, 2, 3, 4]). In this work we will study the problem of exponential stabilization for a class of distributed second order semilinear systems with time delay $r > 0$ described by the following form :

$$\begin{cases} \ddot{x}(t) + Ax(t) + p(t)B\dot{x}(t-r) + f(x(t)) = 0, & t \geq 0, \\ x(t) = x_1(t), \dot{x}(t) = x_2(t), & \forall t \in [-r, 0], \end{cases} \tag{1}$$

where $\dot{\phi} = \dfrac{d\phi}{dt}$ and $\ddot{\phi} = \dfrac{d^2\phi}{dt^2}$. Let H be a real Hilbert space endowed with an inner product $\langle ., . \rangle_H$ and its corresponding norm $\|.\|_H$.

Across this paper, we assume that the following hypotheses hold :

$(\mathcal{H}_1)$: A is a linear (generally unbounded) such that $A = A^* \geq 0$ and coercive on H :

$$\exists \alpha_0 > 0 : \quad \left\| A^{\frac{1}{2}} v \right\|_H^2 \geq \alpha_0 \|v\|_H^2, \text{for all } v \in \mathcal{D}\left(A^{\frac{1}{2}}\right). \tag{2}$$

$(\mathcal{H}_2)$: $B \in \mathcal{L}(H)$.

$(\mathcal{H}_3)$: $f : H \longrightarrow H$ is a nonlinear function, which is supposed globally Lipschitz, namely:

$$\exists L_f > 0 : \quad \|f(u) - f(v)\|_H \leq L_f \|u - v\|_H, \quad \forall u, v \in H, \tag{3}$$

and satisfies $f(0) = 0$ and $\langle f(v), v \rangle_H \geq 0$, $\forall v \in H$.

Furthermore $\mathcal{D}\left(A^{\frac{1}{2}}\right)$ endowed with norm $\|.\|_{\mathcal{D}\left(A^{\frac{1}{2}}\right)}$ such that for any $\phi \in \mathcal{D}\left(A^{\frac{1}{2}}\right)$, $\|\phi\|_{\mathcal{D}\left(A^{\frac{1}{2}}\right)} = \left\| A^{\frac{1}{2}} \phi \right\|_H$ and $X = \mathcal{D}\left(A^{\frac{1}{2}}\right) \times H$ is endowed with the norm $\|.\|_X$ defined by :

$$\|(\phi, \psi)\|_X = \left(\left\| A^{\frac{1}{2}} \phi \right\|_H^2 + \|\psi\|_H^2 \right)^{\frac{1}{2}}, \quad \forall (\phi, \psi) \in X$$

and the following inner product :

$$\langle (v_1, w_1), (v_2, w_2) \rangle_X = \left\langle A^{\frac{1}{2}} v_1, A^{\frac{1}{2}} v_2 \right\rangle_H + \langle w_1, w_2 \rangle_H, \forall (v_1, w_1), (v_2, w_2) \in X.$$

The scalar function $t \longmapsto p(t)$ represents the control. For $t \geq 0$ and $z \in C([-r, +\infty[, H)$, we define $z_t \in \mathcal{C} := C([-r, 0], H)$ by : $z_t(\theta) = z(t + \theta)$, for all $\theta \in [-r, 0]$ and the norm $\|.\|_{\mathcal{C}}$ by :

$$\|z_t\|_{\mathcal{C}} := \sup_{\theta \in [-r, 0]} \|z(t + \theta)\|_H.$$

Similarly, for $t \geq 0$ and $y = (y_1, y_2) \in C([-r, +\infty[, X)$, we define $y_t = (y_1, y_2)_t \in \mathcal{C}_X := C([-r, 0], X)$ by $y_t(\theta) = y(t + \theta) = (y_1(t + \theta), y_2(t + \theta))$, for all $\theta \in [-r, 0]$ and the norm $\|.\|_{\mathcal{C}_X}$ by :

$$\|y_t\|_{\mathcal{C}_X}^2 := \sup_{\theta \in [-r, 0]} \left\{ \left\| A^{\frac{1}{2}} y_1(t + \theta) \right\|_H^2 + \|y_2(t + \theta)\|_H^2 \right\}.$$

Moreover, the corresponding associated free system to (1) is given by :

$$\begin{cases} \ddot{\varphi}(t) + A\varphi(t) + f(\varphi(t)) = 0, & t \geq 0, \\ \varphi(t) = x_1(t), \dot{\varphi}(t) = x_2(t), & t \in [-r, 0]. \end{cases} \tag{4}$$

2 Literature Review

In the case without delay (i.e. $r = 0$), the system (1) is given by :

$$\ddot{x}(t) + Ax(t) + p(t)B\dot{x}(t) + f(x(t)) = 0, \quad t \geq 0. \tag{5}$$

In [6], it has been shown, for $p(t) = 1$ and B is a nonnegative and self-adjoint operator on H, that the following assertions are equivalent :

1. There exist $T_0 > 0$ and $C > 0$ such that every solution of (4) satisfies:

$$\left\| A^{\frac{1}{2}}\varphi(0) \right\|_H^2 + \|\dot{\varphi}(0)\|_H^2 \leq C \int_0^{T_0} \left\| B^{\frac{1}{2}}\dot{\varphi}(t) \right\|_H^2 dt. \tag{6}$$

2. There exist $M > 0$ and $\sigma > 0$ such that for every $(x^1, x^0) \in \mathcal{D}\left(A^{\frac{1}{2}}\right) \times H$, the solution of (5) with $x(0) = x^0$ and $\dot{x}(0) = x^1$ satisfies :

$$\left\| A^{\frac{1}{2}}x(t) \right\|_H^2 + \|\dot{x}(t)\|_H^2 \leq Me^{-\sigma t}\left(\left\| A^{\frac{1}{2}}x^0 \right\|_H^2 + \left\| x^1 \right\|_H^2 \right), \forall t \geq 0 \tag{7}$$

which extended an earlier result in the linear case given in [5].

3 Results and Discussion

The first result concerns the existence and uniqueness of the mild solution to the controlled second order semilinear system with time delay (1), such that B is a bounded linear operator, using the following bounded feedback control :

$$p_r(t) = \rho \frac{\langle B\dot{x}(t-r), \dot{x}(t) \rangle_H}{\|\dot{x}_t\|_{\mathcal{C}}^2} \mathbf{1}_\Theta(t), \quad \text{for all } t \geq 0, \tag{8}$$

where $\rho > 0$ and $\mathbf{1}_\Theta$ is the characteristic function of the set

$$\Theta = \{t \geq 0 : \|\dot{x}_t\|_{\mathcal{C}} \neq 0\}.$$

Theorem 1 : Assuming that the hypotheses $(\mathcal{H}_1) - (\mathcal{H}_3)$ hold. Then, for any $(x_1, x_2) \in \mathcal{C}_X$, there exists a unique global mild solution $x \in C\left([-r, +\infty), \mathcal{D}\left(A^{\frac{1}{2}}\right)\right) \bigcap C^1([-r, +\infty), H)$ of (1) controlled by (8).

Our main results consist on the following theorems, where we give a sufficient and necessary conditions to the exponential stabilization of the system (1).

Theorem 2 (Sufficient condition): Assume that $(\mathcal{H}_1) - (\mathcal{H}_3)$ hold. Further, we assume that there exist $T_1 > r$ and $C > 0$ such that

$$\left\| A^{\frac{1}{2}} \varphi(0) \right\|_H^2 + \|\dot{\varphi}(0)\|_H^2 \leq C \int_r^{T_1} |\langle B\dot{\varphi}(t-r), \dot{\varphi}(t) \rangle_H| \, dt, \tag{9}$$

where $\varphi(t)$ is the solution of (4). Then there exist $\widetilde{M} > 0$ and $\sigma > 0$ such that the solution of the system (1) satisfies :

$$\left\| A^{\frac{1}{2}} x(t) \right\|_H^2 + \|\dot{x}(t)\|_H^2 \leq \widetilde{M} e^{-\sigma t} \left(\left\| A^{\frac{1}{2}} x(0) \right\|_H^2 + \|\dot{x}(0)\|_H^2 \right), \, \forall t \geq 0 \tag{10}$$

using the feedback control (8).

Theorem 3 (Necessary condition): Assume that $(\mathcal{H}_1) - (\mathcal{H}_3)$ hold. If there exist $\widetilde{M} > 0$ and $\sigma > 0$ provided that (10) holds using the feedback control (8). Then, for $\rho > 0$ small, there exist $C > 0$ and $T_1 > r$ such that the solution φ of the system (4) satisfies (9).

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July 27-28, 2023, Faculty of Sciences Casablanca, Hassan II University, Casablanca Morocco

On the Observability for a Class of Linear Time-Fractional Systems.

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Abstract:

In this work, we present the notion of global observability developed for a class of time-fractional systems with a Caputo derivative of order $\alpha \in]1,2[$.This notion is common in the control theory literature and consists of finding and reconstructing the initial state of a given system either over the entire evolution domain (global observability) or only in a given (desired) subregion within it (regional observability) [1, 2, 4] , First, we give definitions and some properties regarding the notion in hand, and then we describe a method for finding the state of the system at t = 0. The method used is an extension of the Hilbert uniqueness method (HUM)[3]. We finish this work with some successful numerical examples to see the effectiveness of the proposed approach.

Keywords: Fractional Calculus, Control Theory, Initial State Reconstruction, HUM Approach, Numerical Approach, Simulations.

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July 27-28, 2023, Faculty of Sciences Casablanca, Hassan II University, Casablanca Morocco

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Simulating Bidimensional Water Infiltration With Radial Basis Functions Methods

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Abstract:
The flow problem into unsaturated porous media is a challenging task to solve due to the nonlinearity of Richards' equation. An accurate solution methodology for all soils, initial and boundary conditions encountered is still pending. This study investigates bidimensional infiltration problems from a surface strip source into homogeneous and heterogeneous soils. We handle the nonlinearity of the governing equations by Gardner's exponential model. Then, we apply a collocation method based on radial basis functions and the partition of unity principle. Numerical tests have demonstrared the numerical behaviour of the scheme, and its applicability to practical unsaturated flow problems.

Keywords: Meshless method, radial basis function, Richards' equation.

1 Introduction

Accurate prediction of water flow into unsaturated porous media is necessary for agriculture, irrigation and water conservation. Richards' equation describing this phenomenon is strongly nonlinear. Thus exact analytical solutions are complex to obtain, hence the need for numerical solutions. In this work, we present a mathematical and numerical study of bidimensional water infiltration problem into an unsaturated soil from a surface strip source of finite width. Radial basis functions (RBF) methods are used to deal with this problem. We apply a radial basis functions stable method combining the QR factorisation algorithm with the partition of unity principle (RBFPUM-QR) [1] to solve the Richards equation. Then we compare the numerical solutions and the analytic solutions available in the literature. Our objective is to show the accuracy of the RBF methods for the problems of infiltration under a strip source and to optimize the results numerically using Matlab tools.

This paper is organized as follows. In section 2, we describe an infiltration problem using Richards equation and the solution procedure applying an RBF method. Then we present our numerical results for the infiltration problem into homogeneous and heterogeneous soils in section 3. The main conclusions and outlooks of this work are reported in section 4.

2 Mathematical model

We discuss two infiltration problems: the first one is water infiltration into homogeneous soil from a surface strip source. In this case, the corresponding Richards equation is

$$C(h)\frac{\partial h}{\partial t} = \frac{\partial}{\partial x}\left(K(h)\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K(h)\frac{\partial h}{\partial y}\right) - \frac{\partial K(h)}{\partial y}, \tag{1}$$

for which the analytic solution of linearized infiltration is presented by Warrick and Lomen [2]. The linearization of equation (1) is obtained by using the Kirchhoff transformation $\phi = \int_{h_0}^{h} K(h)\,dh$ and the Gardner exponential model $K(h) = K_0 e^{\alpha h}$ and $\theta(h) = \frac{K_0}{A_0} e^{\alpha h}$, where K_0, α and A_0 are empirical constants and h_0 is the initial value of h in the soil.

The second problem is water infiltration into heterogeneous soil from a surface strip source. Since the hydraulic properties are considered spatially variable, the Richards equation takes the form

$$C(\underline{x}, h)\frac{\partial h}{\partial t} = \nabla \cdot (K(\underline{x}, h)\nabla h) - \frac{\partial K(\underline{x}, h)}{\partial y}, \tag{2}$$

where $\underline{x} = (x, y) \in \mathbb{R}^2$ is the space variable and the vertical coordinate y is assumed to be positive downward for this case. We linearize the equation using the same techniques as above, besides that, we use the following assumption : $K_s(\underline{x}) = K_0 \exp\{-\lambda_x|x| - \lambda_y y\}$, where λ_x and λ_y are the soil heterogeneity constants. For an infiltrating source of finite width, the deduced equation and its analytic solution are presented by Protopapas and Bras [3].

We express the linear problem in terms of the variable ϕ. Let $X = \{\underline{x}_1, \cdots, \underline{x}_N\}$ be a set of N distinct points belonging to an open set Ω of $\mathbb{R}^2$ and its boundaries. The approximate solution at the time t_n using the global RBF method has the form

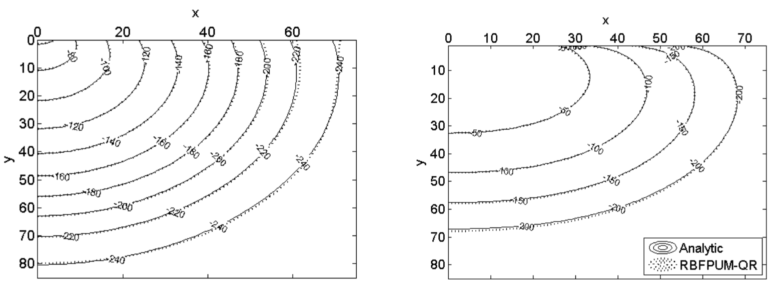
$$\tilde{\phi}^n(\underline{x}) = \sum_{j=1}^{N} \lambda_j^n \psi(||\underline{x} - \underline{x}_j||_2), \ \underline{x} \in \Omega \cup \partial\Omega, \tag{3}$$

where $||.||_2$ is the euclidean norm, $(\lambda_j^n)_j$ are unknown coefficients and ψ is a radial basis function. We consider the Gaussian $\psi(r) = e^{-(\varepsilon r)^2}$ where $r = ||\underline{x} - \underline{x}_j||_2$ and the parameter $\varepsilon > 0$ is called the shape parameter and governs the flatness of the RBF. In order to solve the Richards equation, we apply spacial differential operators to the approximate solution (3). Kansa's method gives accurate results for well-chosen shape parameters. However, the computation cost increases significantly for large node sets. To overcome this issue, we can use the RBFPUM solution based on local RBF approximations [4].

For small values of ε, RBF approximation becomes very accurate, although it forms an ill-conditioned basis. This problem can be managed using the RBF-QR method presented in [1]. Combining this algorithm with the partition of unity principle yields a local method, accurate and stable when $\varepsilon \to 0$ [5].

3 Numerical results

In the following figures, we compare the analytical and numerical pressure head contours at $T = 72min$. We can see that the numerical solutions are very close to the analytical solutions.



(a) Homogeneous soil. (b) Heterogeneous soil in the y direction.

Figure 1: Contours of pressure head for infiltration from a strip source.

4 Conclusion

We employ a numerical method with radial basis functions to solve infiltration problems under a constant flux boundary condition. Due to the spectral accuracy of radial basis functions, locality ensured by the partition of unity principle, and stability guaranteed by the QR-factorization algorithm, we obtain an accurate solution using the RBFPUM-QR method. The convergence of RBF meshless methods encourages us to extend the study to infiltration problems into heterogeneous soils with irregular boundaries.

Acknowledgment

We thank the sponsor and reviewers of the International Conference on Recent Advances in Partial Differential Equations and Applications (ICRAPDEA'23) for this precious opportunity.

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Communication Info

Titre: second Hankel-clifford

Abstract

In this work, we generalize theorem of Hardy for the second Hankel-Clifford transform .f on (0, +∞), such that $\|f\|_{L_{\mu}^{P}} = \int_0^{\infty} |f(x)|^P x^{\mu} dx < \infty$.

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UNCERTAINTY PRINCIPLES FOR LINEAR CANONICAL FOURIER-BESSEL TRANSFORM

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Abstract.

The Linear Canonical Fourier-Bessel transform can be defined as a generalization of the Fractional Fourier-Bessel transform. In this paper we establish the Morgan's theorem for this new transform, we also prove Heisenberg-Weyl's uncertainty principle and hardy's theorem but with different approach.

Keywords: Linear canonical Fourier-Bessel transform; Morgan's UP; Heisenberg-Weyl's UP; Hardy's UP.

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Potential theory for fractional Sobolev space with variable exponents

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Abstract

In this paper we give a finer analysis of fractional variable exponents Sobolev spaces. We use the relative capacity to characterize completely the zero trace fractional variable exponents Sobolev spaces. We also give a relative capacity criterium for removable sets.

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International Conference on Recent Advances in Partial Differential Equations and Applications (ICRAPDEA'23)

July 27-28, 2023, Faculty of Sciences Casablanca, Hassan II University, Casablanca Morocco

On the Stabilization of Prey-Predator Model with Diffusion

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Abstract:
In this paper, we consider a predator–prey model given by a reaction–diffusion system. This model encompasses the classic Holling , Holling , Holling , and Holling functional responses. We investigate the stabilization problem of the considered system using multiplicative controls. By linearizing the system and using the maximum principle, we construct a multiplicative control that exponentially stabilizes the system towards its steady-state solutions. The proposed feedback control allows us to reach a large class of steady-state solutions. The global well-posedness is obtained via Banach fixed point. Applications and numerical simulations to Holling responses I,II, III and IV are presented.

Keywords: : prey predator with diffusion, Holling responses, feedback stabilization, multiplicative controls.

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Forecasting infectious disease transmission with an SEIR model and social contact matrices

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Abstract:
The aim of this study is to examine the impact of intervention and confinement strategies on the spread of COVID-19 using a contact-structured and age-structured susceptible-exposed-infected-recovered (SEIR) model for Morocco. Contact matrices are constructed based on age groups, with confinement and social distancing interventions examined for each age group and location. A significant reduction in virus transmission has been observed when intervention strategies are combined with confinement by age and location (homes, workplaces, schools, shopping centers, and other locations), particularly for superspreaders. This study highlights the importance of implementing age-specific intervention measures and the need to continue social distancing practices after confinement is lifted. These findings are essential for public health policy and provide insight into managing the ongoing COVID-19 pandemic and other respiratory viruses in Morocco.

Keywords: SARS-CoV-2, COVID-19, Contact matrices, Moroccan social contact patterns, Physical distancing.

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July 27-28, 2023, Faculty of Sciences Casablanca, Hassan II University, Casablanca Morocco

Observer Design For A Class Of Discrete Port Hamiltonian Systems

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Abstract:

In this paper, a simple observer design method is presented for a class of discrete-time port Hamiltonian systems. The proposed observer is full order, and it is a copy of the original system dynamics with a corrective term. The suggested design methodology benefits from the port Hamiltonian framework properties. Based on a convenient assumption and exploiting the fact that the observer is structure preserving, the error between the plant and the observer converges exponentially to zero. The key tool in achieving our goal is the contraction analysis method. The observer design method is illustrated in the RLC circuit with some simulation results.

Keywords: Discrete port Hamiltonian systems, Observer design, Contraction analysis.

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An iterative scheme to solve a coupled magnetohydrodynamic flow

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Abstract:
We are concerned with 3D density-dependent magnetohydrodynamics (MHD) system. We consider the model for the flow of two immiscible, incompressible magnetohydrodynamics fluids where the surface tension between both of them is taken into account, Under certain conditions on the data, we show that the existence and uniqueness of the solution of a weak formulation can be guaranteed. We also establish the optimal error estimates for the numerical solution of finite element approximation of the MHD system and present some numerical experiments.

Keywords: coupled magnetohydrodynamic equations, pressure boundary conditions, finite element method, divergence-free approximations.

1 Literature Review

the full, coupled system of time dependant, two immiscible, incompressible magnetchydrodynamics equations in a three-dimensional domain, which describe the unsteady state flow of a viscous, incompressible, electrically conducting fluid. The MHD system has been used as a useful model in the study of magnetic properties of electrically conducting fluids with applications in the study of geophysics, astrophysics, industries and also in engineering such as magnetic propulsion devices, nuclear reactor technology, semiconductor manufacturing, metal hardening, casting, melting and crystal growth (see [Codina and Hernández, 2011] , [Gerbeau et al., 2006]). The majority of the work done on these equations has been for the time-dependent problem with homogeneous boundary conditions. The stationary

MHD equations are treated by M.D. Gunzburger, A.J. Meir and J.S. Peterson in [Gunzburger et al., 1991], where they prove the existence of a solution and its uniqueness in particular cases.
The time-dependent MHD system has been extensively studied by many mathematicians. For instance, with constant density, global existence for standard viscous resistive incompressible MHD system has been previously proved by Duvaut and Lions [Duvaut and Lions, 1972], see also Sermange and Temam [Sermange and Temam, 1983]. Nonhomogeneous boundary conditions for the velocity and magnetic field are treated in [Gerbeau and Le Bris, 1997] where Gerbeau and Le Bris proved existence of a global weak solution.

In addition, many previous studies only consider one or two-dimensional domains see [Gui, 2014]and [Lü et al., 2017], where Lü et al. proved the global existence and large time asymptotic behavior of strong solutions to the nonhomogeneous MHD equations with vacuum, where many effects unique to physical, three-dimensional space are lost; see [Zhang and Yu, 2015]. Also, many studies assume other simplifications to the equations, such as vanishing magnetic Reynolds number, which allows the fluid and magnetic equations to uncouple.

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Projection methods in turbulence

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Abstract:

In this study, we introduce a finite element method for solving the three-dimensional Navier-Stokes equations using the velocity-vorticity-helicity formulation. This formulation directly solves for the variables of velocity, vorticity, Bernoulli pressure, and helical density. To solve this equation, we introduce a family of schemes based on projection methods. Numerical experiments are conducted to validate the expected convergence rates and confirm the effectiveness and accuracy of the proposed schemes.

Keywords: Navier-Stokes equations, velocity-vorticity-helicity formulation, fractional time stepping.

1 Introduction

The Navier-Stokes equations are a fundamental system for the mathematical modeling of fluid flows. They describe the behavior of the fluid in terms of conservation of mass and momentum. Incompressible viscous flows of a Newtonian fluid are modeled by the system of the Navier-Stokes equations, which read:

Let Ω be a bounded domain of $\mathbb{R}^3$ which is either convex or of class $\mathbb{C}^{1,1}$. Let $\partial\Omega = \Gamma_D \cup \Gamma_N$ the boundary of Ω. We denotes by Ω_t the open set $(0,T)\times\Omega$, where $T>0$ is the final time.

$$\left\{\begin{array}{rcll} \frac{\partial u}{\partial t} + (u\cdot\nabla)\,u - \nu\,\Delta u + \nabla\,p & = & f & \text{in } \Omega_t, \\ \nabla\cdot u & = & 0 & \text{in } \Omega_t, \\ u|_{t=0} & = & u_0 & \text{in } \Omega, \\ u & = & 0 & \text{in } \partial\Omega\times(0,T)\,, \end{array}\right. \tag{1}$$

where the unknowns are the velocity u, the pressure p. The positive constant ν is the viscosity of the fluid .The second term in equation (1) f represents

a density of body forces. Some boundary conditions have to be imposed on $\partial\Omega$ to obtain a closed set of equations, we pose the Dirichlet conditions for velocity: $u = 0$ on $\partial\Omega \times (0, T)$
This equation can be equivalente to the following Velocity-Vorticity-Helicity formulation:

$$\begin{cases} \dfrac{\partial u}{\partial t} - \nu\Delta u + w \times u + \nabla P &= f & \text{in } \Omega_t, \\ \dfrac{\partial w}{\partial t} - \nu\Delta w + 2\mathbb{D}(w)\, u - \nabla\eta &= \nabla \times f & \text{in } \Omega_t, \\ \nabla \cdot u &= 0 & \text{in } \Omega_t, \\ \nabla \cdot \omega &= 0 & \text{in } \Omega_t, \\ u|_{t=0} &= u_0 & \text{in } \Omega, \\ w|_{t=0} &= w_0 & \text{in } \Omega, \\ u = w &= 0 & \text{in } \partial\Omega \times (0, T)\,, \end{cases} \quad . \tag{2}$$

The vorticity $w = \nabla \times u$ plays a fundamental role in fluid dynamics as well as in mathematical analysis of the Navier-Stokes equations and in many cases it is advantageous to describe dynamics of a flow in terms of the evolution of the vorticity. P is the Bernoulli pressure variable defined by: $P = \frac{1}{2} u \cdot u + p$ and $\mathbb{D}(w) = \frac{1}{2}\left(\nabla w + \nabla w^T\right)$ the symmetric part of the vorticity gradient and $\eta = u \cdot w$ the helical density.

Our objective is to propose schemes based on projection methods to solve the given equation. Projection methods involve splitting the problem into two main steps: a prediction step and a correction step. In the prediction step, an approximation of the fluid velocity is obtained by solving the advection-diffusion equations, which represent the propagation and diffusion of the fluid's momentum. In the correction step, a correction is applied to the previously computed velocity to satisfy the constraint of a zero divergence velocity field. This is achieved by projecting the computed velocity onto the space of divergence-free vector fields. This projection process ensures mass conservation and provides an accurate approximation of the velocity field. Projection methods are widely used in numerical simulation of fluid flows, particularly for solving complex problems associated with the Navier-Stokes equations.

2 Literature Review

The projection method was initially introduced by Chorin [1] in 1968 . This publication presents the original projection method for incompressible Navier-Stokes equations. It describes the steps of velocity prediction and pressure correction used in the projection method. Since then, many researchers have contributed to the development and improvement of this method. References such as Temam's article in 1979 [2] provided a thorough mathematical analysis of the projection method and laid solid theoretical foundations for its use. It examines the stability and convergence of the method and discusses numerical aspects of its implementation.

The increasing popularity of projection methods can be attributed to several factors. Firstly, these methods allow for the separation of velocity prediction and pressure correction computations, which facilitates numerical implementation. Additionally, they ensure accurate conservation of mass and momentum, which is crucial for reliable results.

Works such as Guermond et al.'s article in 2006 [3] explored different variants of the projection method, including the fractional-step projection method and the orthogonal projection method. These variants have allowed for the solution of specific problems such as free-surface flows and multiphase flows by adapting the basic projection method. Over the years, projection methods have been applied to various applications in fluid mechanics. Studies such as Brown and Cortez's work in 2001 [4] examined the use of projection methods in areas such as computational fluid dynamics, addressing problems such as incompressible flows, turbulent flows, and multiphase flows.

The Velocity Vorticity Helicity (VVH) formulation has been developed to study the kinematic characteristics of turbulent flows, focusing on vorticity and helicity. Researchers have used the VVH formulation to analyze the topology of vorticity lines, study helicity properties, and understand vortex structures in turbulent flows. Works such as Moffatt's in 1969 [5] and Chong et al.'s in 1990 [6] have contributed to the understanding and application of the VVH formulation in the study of turbulent flows. In conclusion, projection methods for the resolution of Navier-Stokes equations have spread and gained popularity due to their efficiency, stability, and accuracy. The research and contributions of many scientists have led to the development and improvement of these methods, establishing solid theoretical foundations and exploring new variants to address specific challenges in flow simulation. These methods continue to be widely used in the fluid mechanics community for various applications.

3 Results and Discussion

After applying the proposed schemes based on projection methods to solve the equation, we obtained the following results and observations:

Accuracy: The projection methods demonstrated good accuracy in approximating the solution of the equation. The schemes effectively reduced the error and provided solutions that closely matched the expected values.

Convergence: The schemes exhibited convergence properties, meaning that as the number of iterations increased, the solutions approached the true solution of the equation. This ensured the reliability and stability of the projection methods.

Computational Efficiency: The projection methods showed efficient performance in terms of computational time and resources required. The schemes effectively leveraged the specific structure of the problem to minimize computational costs, making them suitable for solving large-scale equations.

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International Conference on Recent Advances in Partial Differential Equations and Applications (ICRAPDEA'23)

July 27-28, 2023, Faculty of Sciences Casablanca, Hassan II University, Casablanca Morocco

Non-negative periodic solutions for a degenerate double phase Laplacian parabolic equation

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Abstract:
The goal of this paper is the study of a degenerate parabolic equation with double phase phenomena and strongly nonlinear source under Dirichlet boundary conditions, the existence of a non-negative periodic weak solution is proved. Our proof will be based on the Leray-Schauder topological degree, which presents many issues for this kind of equations, but were overcome by using different techniques or known theorems. The considered system is a possible model for problems where the entity studied has different growth coefficients, p and q in our case, in different areas.

Keywords: Topological degree, Periodic solution, Dirichlet conditions, Generalized Sobolev spaces.

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International Conference on Recent Advances in Partial Differential Equations and Applications (ICRAPDEA'23)

July 27-28, 2023, Faculty of Sciences Casablanca, Hassan II University, Casablanca Morocco

A multi-scale model to assess the effect of temperature and radiation on lettuce growth

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Abstract:
Our study aims to investigate the lettuce plant growth patterns in a field experiment conducted under various environmental circumstances, such as temperature and radiation, at various densities. We combined growth models , such as the Gompertz, Aikman, and Scaife, and Scaife, Cox, and Morris models, with a reaction difusion equation for nutrients concentration. With respect to time, day-degrees, and effective day-degrees, we evaluated palant growth at various densities and during three different seasons of the year.

Simulations reveal that substituting time with degree days and effective day-degrees yielded improved model fit and more precise estimations of growth parameters.

Keywords:Plant growth, multiscale modeling, environmental conditions.

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International Conference on Recent Advances in Partial Differential Equations and Applications (ICRAPDEA'23)

July 27-28, 2023, Faculty of Sciences Casablanca, Hassan II University, Casablanca Morocco

Mathematical Modeling and Analysis of Micropolar Fluid Flow with Frictionless Contact Boundary Conditions

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Abstract:
This study focuses on analyzing the behavior of a nonstationary flow in a domain $\Omega \subset \mathbb{R}^3$ using partial differential equations. Specifically, we investigate the dynamics of an incompressible Newtonian micropolar fluid. The micropolar fluid model considered in this research incorporates frictionless boundary conditions for the velocity field and Neumann conditions for the micro-rotational velocity. To address the problem, we derive a variational formulation consisting of a coupled system. This system comprises a nonlinear variational equation for the velocity field and a linear variational equation for the angular velocity. Utilizing fundamental mathematical principles, such as the Galerkin method for variational equations, the Picard-Lindelöf theorem, and the fixed-point theorem, we establish the existence of a unique weak solution for the evolution equations governing the incompressible micropolar fluid model.

Keywords. Micropolar flow model; Incompressible Newtonian flow; Frictionless boundary conditions; Weak solution; Fixed point theorem.

1 Introduction

Researchers have acknowledged the limitations of classical fluid theory in describing complex fluids, such as polymeric fluids, liquid crystals, colloidal systems, suspensions, and emulsions. These limitations arise from the failure to account for the local structure and microrotation of fluid elements, particularly at micro- and nanoscales [1]. To overcome these limitations, Eringen, in collaboration with Suhubi, developed the theory of micropolar fluids [2, 3, 4]. Micropolar fluids can be conceptualized as fluids composed of rigid, randomly oriented, or spherical particles suspended within a viscous medium.

In this study, we concentrate on non-stationary incompressible Newtonian micropolar fluids with frictionless boundary conditions for the velocity field and homogeneous Neumann boundary conditions for the microrotational velocity. Our objective is to investigate the application of this model to a physical process that incorporates the microstructure of the fluid. Consequently, we examine the system of partial differential equations that describes this process within the domain Ω. Our main focus is to demonstrate that this model leads to a well-posed mathematical problem and establish the existence and uniqueness of weak solutions.

2 Problem statement and variational formulations

The physical setup is as follows: We consider an open, bounded, and connected set Ω in $\mathbb{R}^d$, where $d = 2$ or 3, which is filled with a viscous, incompressible, Newtonian micropolar fluid. The boundary of Ω is denoted by $\Gamma = \partial\Omega$. The time interval of interest is $t \in (0, T)$, where $0 < T < \infty$. Additionally, we assume that the boundary Γ can be divided into two sets, $\bar{\Gamma}_1$ and $\bar{\Gamma}_2$, with disjoint relatively open subsets Γ_1 and Γ_2, where the measure of Γ_1 is greater than zero. In this study, we analyze a coupled nonlinear system of partial differential equations that describes the balance of momentum, angular momentum, and mass within the space-time region $Q = \Omega \times (0, T)$ (see [5] for further details).

The system of equations is as follows:

$$\begin{cases} \rho_0 \dfrac{\partial u}{\partial t} + \rho_0 (u \cdot \nabla) u + \nabla p - 2(\nu + \nu_r)\nu_1 \operatorname{div}(D(u)) = 2\nu_r \ \operatorname{curl}(\omega) + f, & \text{in } Q, \\ \begin{array}{l} J_0 \rho_0 \frac{\partial \omega}{\partial t} + J_0 \rho_0 (u \cdot \nabla)\omega - (c_d + c_a)\Delta(\omega) - (c_0 + c_d - c_a)\nabla(\operatorname{div}(\boldsymbol{\omega})) + 4\tau\omega \\ = 2\nu_r \ \operatorname{curl}(u) + g, \end{array} & \text{in } Q, \\ \operatorname{div}(u) = 0, & \text{in } Q. \end{cases}$$

Here, the variables $u := u(x, t) \in \mathbb{R}^3$, $w := w(x, t) \in \mathbb{R}^3$, and $p := p(x, t) \in \mathbb{R}$ represent the linear velocity field, the velocity of rotation of the particles, and the fluid pressure at the point $(x, t) \in Q$, respectively. The symmetric part of the velocity gradient, denoted by $D(u)$, is given by $D(u) = \frac{1}{2}[\nabla u + (\nabla u)^T]$, where S^d represents the space of second-order symmetric tensors on $\mathbb{R}^d$. The functions f and g are predetermined and represent external sources of linear and angular momentum of particles, respectively. The positive real constants ν, ν_r, c_0, c_a, and c_d characterize the isotropic properties of the fluid. Specifically, ν is the Newtonian viscosity, ν_r is the viscosity of microrotation, and c_0, c_a, and c_d are viscosities related to the asymmetry of the stress tensor, satisfying the condition $c_0 + c_d > c_a$. For simplicity, we denote $\nu_1 := 2(\nu + \nu_r)$, $\nu_2 := c_d + c_a$, and $\nu_3 := c_0 + c_d - c_a$. Without loss of generality, we assume the density of the fluid to be equal to one. Additionally, we denote $div(D) = (\nabla \cdot D)$ as the divergence operator for vector-valued functions.

The mathematical formulation of the micropolar fluid flow model for incompressible Newtonian fluids can be stated as follows:

Problem P**:** *Find a velocity field* $u : \Omega \times (0, T) \to \mathbb{R}^d$ *and an angular velocity field* $\omega : \Omega \times (0, T) \to \mathbb{R}^d$ *satisfying the following equations:*

$$
\begin{aligned}
&\frac{\partial u}{\partial t} + (u \cdot \nabla)u + \nabla p + 2c_a \operatorname{div}(D(u)) = 2\nu_r \operatorname{curl}(\omega) + f && \text{in } Q, && (2.1)\\
&\frac{\partial \omega}{\partial t} + (u \cdot \nabla)\omega - \nu_2 \Delta\omega - \nu_3 \nabla(\operatorname{div}(\omega)) + 4\tau\omega = 2\nu_r \ \operatorname{curl}(u) + g && \text{in } Q, && (2.2)\\
&\operatorname{div}(u) = 0 && \text{in } Q, && (2.3)\\
&D(u)_\tau = 0, \quad u_\nu = 0, \quad \frac{\partial \omega}{\partial n} = 0 && \text{on } \Gamma_2 \times (0,T), && (2.4)\\
&u = 0, \quad \omega = 0 && \text{on } \Gamma_1 \times (0,T). && (2.5)
\end{aligned}
$$

In this formulation, the problem seeks to determine the velocity field u and the angular velocity field ω satisfying the equations (2.1)-(2.5). Equation (2.1) represents the momentum balance equation, taking into account the convective and pressure terms, as well as the effects of microrotation and external forces f. Equation (2.2) governs the evolution of the angular velocity field ω and incorporates the Laplacian and gradient terms, along with the effects of microrotation and external sources g. The incompressibility condition is expressed by equation (2.3), which requires the divergence of the velocity field u to be zero. The boundary conditions (2.4) impose the tangential derivative of the velocity field to be zero, the normal component of the velocity to vanish, and the normal derivative of the angular velocity to be zero on the boundary Γ_2 over the time interval $(0,T)$. Finally, the conditions (2.5) enforce the velocity field u and the angular velocity field ω to be zero on the boundary Γ_1 over the time interval $(0,T)$.

Now, we introduce several spaces to establish the variational formulation. Let us define the spaces as follows: $\tilde{U} = \{u \in C_0^\infty(\Omega), \ \operatorname{div} = 0 \ \text{in} \ \Omega, \ u = 0 \ \text{on} \ \Gamma_1, \ u_\nu = 0 \ \text{on} \ \Gamma_2\}$, $U =$ closure of $\tilde{U}$ in $H^1(\Omega)$, $\quad H =$ closure of $\tilde{U}$ in $L^2(\Omega)$, and $\quad W = \{\omega \in L^2(\Omega), \ \operatorname{curl}(\omega) \in L^2(\Omega), \ \omega = 0 \ \text{on} \ \Gamma_1\}$. The space W is equipped with the norm $\|.\|_{H^1(\Omega,\mathbb{R}^d)}$, and U is equipped with two norms: $\|u\|_U = \|u\|_{W^{1,p}(\Omega,\mathbb{R}^d)}$ and $\|v\|_U = \|D(v)\|_{L^p(\Omega,\mathbb{S}^d)}$ for all $u \in U$ and $v \in U$, respectively. Here, p is a positive constant, and $D(v)$ denotes the symmetric gradient of v. The norms $\|.\|_{W^{1,p}(\Omega,\mathbb{R}^d)}$ and $\|.\|_U$ are equivalent on U, as implied by the Korn inequality. Moreover, U is a reflexive separable Banach space, H is a separable Hilbert space, and the embedding $U \subset H$ is continuous and dense. Thus, $(U, H, U^\star)$ forms an evolution triple of spaces. In this setting, H is identified with its dual, and we have $U \subset H \subset U^\star$ with dense and continuous embeddings.

Finally, we define the spaces $\mathcal{H}$, $\mathcal{H}^\star$, $\mathcal{U}$, and $\mathcal{W}$ as follows: $\mathcal{H} = L^2(0,T;W)$, $\mathcal{H}^\star = L^2(0,T;W^\star)$, $\mathcal{U} = \{u \in L^2(0,T;U) \mid u' \in L^2(0,T;U^\star)\}$, $\mathcal{W} = \{\omega \in \mathcal{H} \mid \omega' \in \mathcal{H}^\star\}$.

The variational formulation of problem (2.1)-(2.5) is as follows:

Problem PV: *Find a velocity field $u \in \mathcal{U}$ and an angular velocity field $\omega \in \mathcal{W}$ such that*

$$
\begin{aligned}
&\int_\Omega \frac{\partial u}{\partial t} \cdot v, \mathrm{d}x + \int_\Omega (u \cdot \nabla)u \cdot v, \mathrm{d}x + \int_\Omega D(u) : D(v), \mathrm{d}x - 2\tau \int_\Omega \operatorname{curl}(\omega) \cdot v, \mathrm{d}x \\
&= \int_\Omega f \cdot v, \mathrm{d}x \quad \forall v \in U,
\end{aligned} \tag{2.6}
$$

$$
\begin{aligned}
&\int_\Omega \frac{\partial \omega}{\partial t}\varphi, \mathrm{d}x + \int_\Omega (u\cdot\nabla)\omega\cdot\varphi, \mathrm{d}x - c_d\int_\Omega \mathrm{curl}(\omega)\cdot\mathrm{curl}(\varphi), \mathrm{d}x \\
&\qquad + (c_d + c_b)\int_\Omega \nabla\omega\cdot\nabla\varphi, \mathrm{d}x + 4\tau\int_\Omega \omega\varphi, \mathrm{d}x - 2\tau\int_\Omega \mathrm{curl}(u)\cdot\varphi, \mathrm{d}x \qquad (2.7)\\
&= c_d\int_{\Gamma_2}\mathrm{curl}(\omega)\cdot\varphi, \mathrm{d}x + \int_\Omega g\varphi, \mathrm{d}x \quad \forall\varphi\in W.
\end{aligned}
$$

This formulation is obtained by using arguments similar to those in [6].

The existence and uniqueness of the solution (u, ω) satisfying (2.6)-(2.7) can be established using the Galerkin method and the Cauchy-Lipschitz theorem.

Conclusion: In this work, we have investigated the behavior of a viscous, incompressible, Newtonian micropolar fluid using a specific model. We have considered a flow scenario without friction in the velocity field and assumed homogeneous Neumann boundary conditions for the microrotational velocity. By formulating the problem variationally, we have demonstrated the existence of a weak solution by employing the Galerkin method and the Cauchy-Lipschitz theorem. Furthermore, we have established the uniqueness of the solution under an additional smallness assumption on the problem data.

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Comparative analysis of previous works on landslides analysis

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Abstract:
Landslides are frequent worldwide events that can be triggered by natural or human actions or both combined and can lead to economic and social damages of different degrees. Either way, they should be predicted in advance in order to bring the predictive measures at the appropriate time, or the right corrective measures if the predictive ones weren't possible in a certain timeframe. This paper is based on a first comparative analysis of two previous thesis papers related to landslides analysis, we'll explore the methodologies and approaches used as well as obtained results.

Keywords: Landslides, Landslides hazard, Landslides hazard assessment, landslides susceptibility maps, landslides hazard maps, landslides risk maps. .

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Top Machine Learning Approaches Recommended for Sentiment Analysis : Moroccan Use Case

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Abstract:

Sentiment analysis or opinion mining refers to the set of operations aimed at extracting emotions from a given dataset and subsequently classifying them into categories, initially as positive or negative. To enhance our results, we can expand the classification to include additional categories such as positive, negative, and neutral.

Sentiment analysis has been implemented in various fields, including politics and business, enabling significant decision-making capabilities.

This paper will focus on the methodology to be followed, starting from different preprocessing techniques to training with a classification algorithm.

Our objective in this paper is to develop a sentiment analysis model based on classification algorithms using the MAC dataset. Additionally, we will compare the most commonly used classifiers in similar cases based on their accuracy.

Keywords: NLP, opening mining, ML, ASA, MAC, SA.

1 Introduction

Sentiment analysis falls under the field of text mining or text analysis. This analytical technique involves extracting the meaning from various textual sources, such as survey responses, online reviews, or social media comments. Sentiments expressed in the text are then assigned a score, typically -1 for negative and +1 for positive, using natural language processing (NLP). Given the vast number of publications shared on the internet, particularly on social media platforms, sentiment analysis helps companies monitor the perception of their brand or products at any given moment [1].

While most studies have primarily focused on English, other languages like Arabic have received less attention, despite the increasing volume of Arabic comments on the internet, especially those in dialect [2].
This paper begins by highlighting the need for further investment in this area, as the number of works conducted on sentiment analysis for Arabic, particularly Moroccan dialect. We implement popular machine learning approaches recommended for Arabic sentiment analysis, including SVM, which has shown promising results with an accuracy of over 82% in previous works [2].
Likewise [3] confirmed and approved by they work for opinions on health services using a dataset collected from Twitter in which they experimented various Machine and Deep learning approaches, their experiments were based on SVM, Naive Bayes and CNNs. The best classifier with outstanding results was SVM with 91% accuracy. In this paper we implement MAC (Moroccan Arabic Corpus) to make a sentiment analysis based on comments written in Moroccan dialect.

In this paper we implement MAC (Moroccan Arabic Corpus) [4] to make a sentiment analysis based on comments written in Moroccan dialect. Our contributions in this work can be summarized as:

- Investigating and describing the properties of the MAC dataset.
- Presenting preprocessing techniques for MAC comments in preparation for sentiment analysis.
- Conducting a series of experiments on the dataset to compare different classifiers.

2 DATASET

The MAC (Moroccan Arabic Corpus) is the first and largest open-source dataset specifically designed for sentiment analysis in Moroccan dialect. It stands out due to its extensive collection of tweets, which have been manually labeled as positive, negative, or neutral [4] .
The dataset is characterized by the following:
• Open source: The MAC dataset is freely available for public use and can be accessed by anyone.
• More than 18,000 manually labeled tweets: The dataset contains a substantial number of tweets that have been annotated with sentiment labels by human annotators.
• A lexicon-dictionary of 30,000 words: The MAC dataset includes a comprehensive lexicon or dictionary consisting of approximately 30,000 words.
• Mainly composed of three fields: The dataset is primarily composed of three fields, namely tweets, type, and class. These fields provide

essential information about the text, its source, and its sentiment classification.

3 EXPERIMENTATION AND RESULTS

At this stage, our objective is to compare the performance of the most commonly used machine learning algorithms for addressing sentiment analysis tasks in the Arabic language. These algorithms include:

• Support Vector Machine (SVM): SVM is a powerful and widely utilized classification algorithm that aims to find an optimal hyperplane to separate different classes. It has shown promising results in sentiment analysis tasks.

• Naive Bayes Classifier (NB): NB is a probabilistic classifier that applies Bayes' theorem with the assumption of independence among features. It is known for its simplicity and efficiency in text classification tasks, including sentiment analysis.

• k-Nearest Neighbors (KNN): KNN is a non-parametric algorithm that classifies data points based on their proximity to the k nearest neighbors. It is a versatile algorithm that can be applied to various classification problems, including sentiment analysis.

By comparing these algorithms, we can evaluate their effectiveness and determine which one performs best for sentiment analysis in the Arabic language.

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Numerical Analysis and Stability of the Moore-Gibson-Thompson-Fourier Model

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Abstract:
In this paper, we consider the Moore-Gibson-Thompson-Fourier Model. Our contribution will consist in studying the numerical stability of the Moore-Gibson-Thompson-Fourier system. First we introduce a finite element approximation after the discretization, then we prove that the associated discrete energy decreases and later we establish a priori error estimates. Finally, we obtain some numerical simulations.

Keywords: Moore-Gibson-Thompson-Fourier Model, numerical stability, finite element method, numerical simulations.

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Some stationaries and non-stationaries hidden Markov chain : application in imagery

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Abstract:
In this work, we propose three applications of textured image segmen- tation using the hidden Markov chain. In the first one, we use the classical model Hidden Markov Chain(HMC) to segment images. In the second one, we segment them by the model Pairwise Markov Chain(PMC). And we use Triplet Markov Chain(TMC) in the last one. Our purpose is to discover the most performant model that can segment image under any condition like noise, textured.....
The presented results confirm that Triplet Markov Chain is the best.

Keywords: HMC, PMC, TMC, Stationnary process, Non-stationnary process Textured image segmentation.

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International Conference on Recent Advances in Partial Differential Equations and Applications (ICRAPDEA'23)

July 27-28, 2023, Faculty of Sciences Casablanca, Hassan II University, Casablanca Morocco

A fractional epidemiological model of measles transmission

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Abstract:

The primary objective of this research article is to enhance the existing measles transmission epidemic model by incorporating the fractional order derivative of Caputo. The analysis focuses on investigating the local and global stability of the equilibrium through the utilization of a potential Lyapunov function. Additionally, the study discusses the inclusion of the fractional optimal control problem related to control strategies. Furthermore, numerical simulations are conducted to illustrate the stability of equilibria and to analyze the behavior of the obtained solutions.

Keywords: Caputo derivative, Dynamical systems, Stability analysis, Reproduction number.

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International Conference on Recent Advances in Partial Differential Equations and Applications (ICRAPDEA'23)
July 27-28, 2023, Faculty of Sciences Casablanca, Hassan II University, Casablanca Morocco

On A Singular Elliptic Problem

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Abstract:
In this work, we show the existence and uniqueness of solution to the following problem:

$$\begin{cases} -\Delta u &= \frac{|\nabla u|^p}{|u|^a} + \lambda f \quad \text{in} \quad \Omega \\ u &= 0 \quad\quad\quad\quad\ \text{on} \quad \partial\Omega \end{cases} \tag{1}$$

where $N > 2$, Ω is a bounded regular open subset of $\mathbb{R}^N$, $p > 2$, $a < \frac{p}{2}$, $\lambda > 0$ and f is a nonnegative function belonging to a suitable Lebesgue space.

Keywords: singular elliptic problem, super-quadratic gradient, fixed point arguments, β-convex set.

1 Introduction

The main goal of this work is to study existence for the singular elliptic problem with homogeneous Dirichlet condition

$$\begin{cases} -\Delta u &= \frac{|\nabla u|^p}{|u|^a} + \lambda f \quad \text{in} \quad \Omega \\ u &= 0 \quad\quad\quad\quad\ \text{on} \quad \partial\Omega \end{cases} \tag{2}$$

where $N > 2$, $\Omega \subset \mathbb{R}^N$ is a bounded regular domain, $p > 2$, $a < \frac{p}{2}$, $\lambda > 0$ and $f \in L^m(\Omega), m \geq 1$ is a nonnegative function.
In ordre to solve problem (2), we established a Sobolev regularity for $v^{1-\sigma}$, where v is the unique solution for the Poisson problem and $\sigma < \frac{1}{2}$.

Under additional hypotheses on the data, we were able to show that our solution is unique.

Since $a < \frac{p}{2}$, then problem (2) can be written as the following problem

$$\begin{cases} -\Delta u &= \frac{1}{(1-\sigma)^p}|\nabla u^{1-\sigma}|^p + \lambda f \quad \text{in} \quad \Omega \\ u &= 0 \quad\quad\quad\quad\quad\quad\quad\quad\quad\ \text{on} \quad \partial\Omega \end{cases} \tag{3}$$

where $\sigma = \frac{a}{p} < \frac{1}{2}$.
These results are part of the paper [6].

2 Literature Review

In the literature, elliptic problems involving gradient terms have received considerable attention. In the absence of the singular term, the previous issue can be viewed as the stationary case of the well-known Kardar-Parisi-Zhang equation that describes many physical phenomena such as growth and roughening surface, propagation of flames , see [10],[9], [11].

While considering the singular term and with $q \leq 2$, the authors in [7, 3, 1, 2] had prove some existence results. In [7] for the problem

$$-\Delta u + \alpha u = \frac{|\nabla u|^2}{u^a} + f(x)$$

with $\alpha > 0$, $a > 0$, $f \in L^\infty(\Omega)$. The existence of solution was proved in $W^{1,2}_{loc}(\Omega) \cap L^\infty(\Omega)$ for $a \geq 1$.

In [1], the authors take into account the larger problem

$$\begin{cases} -\Delta u &= \frac{|\nabla u|^q}{u^{q\mu}} + \lambda f(x) \quad \text{in } \Omega \\ u &= 0 \qquad\qquad\qquad \text{on } \partial\Omega, \end{cases} \tag{4}$$

with $q \leq 2$, then if $q\mu > 1$, they proved the existence of a weak solution for all $f \in L^1(\Omega)$.

3 Another section

We recall the Poisson problem

$$\begin{cases} -\Delta w &= g \quad \text{in} \quad \Omega \\ w &= 0 \quad \text{in} \quad \partial\Omega \end{cases} \tag{5}$$

where $N > 2$, $\Omega \subset \mathbb{R}^N$ is a bounded regular domain and $g \in L^m(\Omega)$ such that $m \geq 1$.

Now we define the notion of weak solution.

Definition. Let $g \in L^1(\Omega)$. We say that $w \in L^1(\Omega)$ is a *weak solution* to (5) if

$$\int_\Omega w(-\Delta\varphi)\,dx = \int_\Omega g(x)\varphi\,dx, \tag{6}$$

for all $\varphi \in \mathcal{C}_0^\infty(\Omega)$.

In order to investigate the Poisson problem, the next fundamental theorem is required. For more details one can check [4], [5] and [8].

Proposition *Assume that $g \in L^1(\Omega)$, then problem (5) has a unique weak solution w, such that*

$$w \in L^\theta(\Omega)\,, \quad \forall\, \theta \in [1, \frac{N}{N-2}) \tag{7}$$

and

$$|\nabla w| \in L^r(\Omega), \quad \forall\, r \in [1, \frac{N}{N-1}). \tag{8}$$

Moreover, for $p \in [1, \frac{N}{N-1})$ fixed, the operator $\hat{T}\,:\ L^1(\Omega) \to W_0^{1,p}(\Omega)$ defined by $\hat{T}(g) = w$ where w solves (5) is continuous and compact.

4 Results and Discussion

We state the main regularity result for the Poisson problem (5).

Theorem. *Suppose that $g \in L^m(\Omega)$, $m \geq 1$ is such that $g \gneq 0$ in Ω. Let w be the unique solution of the Poisson problem (5). Then for all $0 < \sigma < \frac{1}{2}$*

$$w^{1-\sigma} \in W_0^{1,p}(\Omega), \qquad \forall\, p < p^*$$

where

$$p^* := \begin{cases} \frac{mN}{(N-m)(1-\sigma)+Nm\sigma}, & \text{if } m < N, \\ \frac{1}{\sigma}, & \text{if } m \geq N. \end{cases} \tag{9}$$

Moreover, There exists a positive constant $C = C(N, m, p, \sigma, \Omega)$ such that

$$\|w^{1-\sigma}\|_{W_0^{1,p}(\Omega)} \leq C\|g\|_{L^m(\Omega)}^{1-\sigma} \tag{10}$$

Now, let us present our main existence result.

Theorem *Assume that $f \in L^m(\Omega), m \geq 1$ is a nonnegative function. Then,*

1. *If* $\frac{N}{2} < m < N$ *and let* p, σ *such that* $2 < pm < \frac{Nm}{(N-m)(1-\sigma)+Nm\sigma}$, *then for* λ *small, there exists a weak solution* u *to the problem 3 such that* $u^{1-\sigma} \in W_0^{1,q}(\Omega)$ *for all* $1 \le q < \frac{mN}{(N-m)(1-\sigma)+Nm\sigma}$.

2. *If* $m \ge N$ *and let* p, σ *such that* $2 < pm < \frac{1}{N\sigma}$, *then for* λ *small, there exists a unique weak solution* u *to the problem 3 such that* $u^{1-\sigma} \in W_0^{1,q}(\Omega)$ *for all* $1 \le q < \frac{1}{\sigma}$.

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On Some Nonlocal Parabolic Systems With Gradient Source Terms

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Abstract:
Within this conference, we will present some results about existence, nonexistence and blow up in finite time $T^* > 0$, of solutions to the following System:

$$\begin{cases} \partial_t u + (-\Delta)^s u &= |\nabla v|^q + f & \text{in} & \Omega \times (0,T), \\ \partial_t v + (-\Delta)^s v &= |\nabla u|^p + g & \text{in} & \Omega \times (0,T), \\ u(x,t) = v(x,t) &= 0 & \text{in} & (\mathbb{R}^N \setminus \Omega) \times (0,T), \\ u(x,0) &= u_0(x) & \text{in} & \Omega, \\ v(x,0) &= v_0(x) & \text{in} & \Omega, \end{cases} \tag{1}$$

where Ω is a bounded regular open subset of $\mathbb{R}^N$, $N > 2s$, $\frac{1}{2} < s < 1$, p, $q \geq 1$, f, g, u_0 and v_0 are measurable nonnegative functions satisfying some appropriate assumptions. The operator $(-\Delta)^s$ is the classical fractional Laplacian defined by

$$(-\Delta)^s u(x) := a_{N,s} \text{ P.V.} \int_{\mathbb{R}^N} \frac{u(x) - u(y)}{|x-y|^{N+2s}} \, dy, \quad s \in (0,1), \tag{2}$$

where PV is the principle value and $a_{N,s}$ is a normalization constant.

As we have mentioned above, our study consists to analyse under some suitable hypothesis on the data, the question of existence, nonexistence as well as explosion of solutions. To this end, we went through the argument of the regularity analysis of the linear equation corresponding to (S) and through the Schauder fixed point theorem.

This work is a collaborative effort with K. Biroud, E.-H. Laamri and M. Daoud.

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On LMI Conditions To Design observer-based control for linear systems with uncertain parameters

Hassan II University, Casablanca Morocco

Authors:

Hiba Hizazi

Mustapha Lhous

Abstract:

It is well known that in many practical control systems, the system often presents some uncertainties and perturbations may be due to additive unknown internal or external noise, environmental influence, nonlinearities, data errors, etc.

In many real models, state feedback control might fail to guarantee the stabilizability when some of the system states are not measurable. Observer-based controllers are often used to stabilize unstable systems or to improve the system performances.

We propose to design observers for a class of uncertain linear systems by using LMI techniques. The observer design is formulated as an LMI feasible problem which easily solved by standard convex optimization algorithms. We give an example to illustrate the proposed results.

Keywords:

(1) Observer-based control
(2) Linear Matrix Inequalities
(3) Uncertain linear system
(4) Lyapunov function
(5) Stability

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Domination In Linear Fractional Order Disturbed System

Hassan II University, Casablanca Morocco

Authors:

Hiba Hizazi
Mustapha Lhous

Abstract:

In this work, we examine the notion of domination for fractional order linear disturbed systems for finite dimensional state.

This consists in studying the possibility to make a comparison between input operators, with respect to the output one, and we give characterization results.

The relationship of controllability and classification is also given and we provide some examples to illustrate our results.

Keywords:

(1) Dynamical system
(2) Domination
(3) Fractional order

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The behavior of a stochastic system

Abstract

In this work, we explore a stochastic model and show that this model has a unique global positive solution that belongs to a positively invariant Set. Then by stochastic Lyapunov functional methods, we investigate the asymptotic behavior of this solution.

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Well-Posedness and Optimal Control of a Parabolic System with Nonlinear Diffusion Terms Modeling the Fast Spread of an Epidemic

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Abstract:
In this talk, we deal with an optimal control problem for a spatio-temporal epidemic model with density-dependent diffusion terms. First, due to the high-ordered coupled terms, we present an approach allowing to deal with the well-posedness of the considered model, by relying on Schauder fixed-point theorem and a regularization technique. Then, based on dual arguments, we establish and characterize an optimal pair for the considered optimization problem. At last, we present some numerical simulations to illustrate the effectiveness of the considered optimal control strategy.

Keywords: Partial differential equations, Optimal control, Epidemic model.

1 Introduction

Throughout the decades, mathematical modeling has proven to be of great use when it comes to the prediction of epidemics. In fact, due to the pioneering work of Kermack and McKendrick in the framework of compartmental modeling, numerous epidemic models have been proposed and analyzed. The mathematical analysis of a given epidemic model can be conducted in order to deal with the following major problems:

- The mathematical well-posedness and biological feasibility: In this regard, one has to establish that for any given set of input parameters, initial state and time-horizon, there exists a unique solution that describes the evolution of the epidemic within the studied population.

- The optimal control problem: When the input parameters no longer satisfy the conditions guaranteeing the convergence of the dynamics towards a free-disease state, one is faced with the question of acting on the population, in order to steer it, at a minimal cost, to a state where the epidemic no longer persists. Mathematically, this can be formulated as an optimal control problem.

In the framework of ordinary differential equations or reaction-diffusion equations, the problems of well-posedness and optimal control for epidemic models are now well-understood and numerous results have been shown, which clearly illustrate the effectiveness of vaccination and treatment when it comes to controlling the spread of an epidemic. However, all the established results are based upon a simplifying assumption stating that the diffusion rate of individuals doesn't depend on the current state of the population, which is not practical. Consequently, the aim of this work is to mathematically incorporate this effect by considering a more complex class of epidemic models [5].

2 Literature Review

To outline the main novelties of our work, we recall some existing relevant works. In [1], Zhou *et al.* considered a reaction-diffusion SIR model with constant diffusion rates with the introduction of two control variables: vaccination and treatment. For fixed controls, the existence and uniqueness of the solution to the state problem was derived by means of semi-group theory. Then, in order to minimize the densities of the susceptible and infected populations at a minimal cost, a suitable cost functional was considered, and necessary optimal conditions for its minimization were derived. The same types of results were proven by Adnaoui *et al.* [2] for an SEIR model with homogeneous spatial diffusion variables, which is an extension of the SIR model with the consideration of the exposed class, in order to incorporate the incubation period. By introducing the same type of control variables, the authors proved the mathematical and biological well-posedeness of the considered model by relying on semi-group theory. Then, in order to minimize the densities of the susceptible and infected populations and in the aim of maximizing the density of the recovered population at a minimal cost, a modified cost functional was considered. In [3], Dai and Liu considered a reaction-diffusion predator prey model with disease in prey, to which they added a control variable representing the dose of medicines given to the infected prey population. By means of semi-group theory, the existence of a unique positive solution to the model was proved. Additionally, in order to minimize the density of the infected prey at a minimal cost, an adequate cost functional was considered and first order necessary optimal conditions were provided. In [4], Miyaoka *et al.* considered a reaction-diffusion model of type SIR, in order to model the propagation of the Zika virus. Contrarily to the previous works, heterogeneous spatial diffusion rates were considered and the existence and uniqueness of the solution to the model was proven in the weak sense. Moreover, in order to incorporate the role of vaccination, the authors considered a control variable representing the vaccination of the susceptible population. Moreover, in the aim of minimizing the densities of the susceptible and infected populations at a minimal cost of vaccine, a suitable cost functional was chosen.

3 Statement of the problem

Let $\Omega \subset \mathbb{R}^N$ ($N \in \{1,2,3\}$) be an open bounded set with boundary $\partial\Omega$ of class C^2 and $\overrightarrow{n}$ the outward normal vector on $\partial\Omega$. Furthermore, let $T > 0$ and denote by $Q_T := \Omega \times (0,T)$ the space-time cylinder with boundary $\Sigma_T := \partial\Omega \times (0,T)$. Given $L, M \in (L^\infty(Q_T))^+$, $\sigma > 0$, we are interested in the minimization of the following functional:

$$J(u,v) := \int_{Q_T} (L(x,t)S(x,t)+M(x,t)I(x,t))dxdt+\frac{\sigma}{2}\left(\|u\|^2_{L^2(Q_T)} + \|v\|^2_{L^2(Q_T)}\right),$$

which is defined on the following set of admissible controls

$$\mathcal{U}_{ad} := \left\{(u,v) \in (L^2(Q_T))^2 \mid u(x,t), v(x,t) \in [0,1] \text{ almost everywhere } \forall (x,t) \in Q_T\right\},$$

and is subjected to the following constraints:

$$\begin{cases} \partial_t S - \boldsymbol{\nabla} \cdot (a_1(S,I,R)\nabla S) = \Lambda - g(S,I) - (\mu + u)S & \text{in } Q_T, \\ \partial_t I - \boldsymbol{\nabla} \cdot (a_2(S,I,R)\nabla I) = g(S,I) - (\mu + v + \gamma)I & \text{in } Q_T, \\ \partial_t R - \boldsymbol{\nabla} \cdot (a_3(S,I,R)\nabla R) = \gamma I - \mu R + uS + vI & \text{in } Q_T, \end{cases} \tag{1}$$

equipped with the following homogeneous Neumann boundary conditions

$$a_1(S,I,R)\nabla S.\overrightarrow{n} = a_2(S,I,R)\nabla I.\overrightarrow{n} = a_3(S,I,R)\nabla R.\overrightarrow{n} = 0 \quad \text{on } \Sigma_T, \tag{2}$$

and the positive initial conditions

$$S(.,0) := S_0 \geq 0,\ I(.,0) := I_0 \geq 0,\ R(.,0) := R_0 \geq 0 \quad \text{in } \Omega, \tag{3}$$

In the above system, g is the incidence function and $a_i (i \in \{1,2,3\})$ denote the density-dependent diffusion terms.

4 Results and Discussion

The results of this work are briefly summarized as follows. We have considered a new spatio-temporal SIR epidemic model with nonlinear diffusion terms and with a general class of incidence functions. To incorporate the role of vaccination and treatment, we have introduced two control variables to the model. Then, for fixed controls, thanks to Schauder fixed point theorem, the mathematical as well as biological well-posedness of the model were proved, which in turns allowed us to prove the existence of an optimal solution to the optimal control problem. Moreover, by proving the weak $\mathcal{G}$-differentiability of the control-to-state mapping, an adjoint problem was derived, which once solved together with the state problem (1)-(3), gives a characterization of the optimal controls. It is noteworthy that there are still some important research questions in regards to the new-proposed epidemic model. For instance:

- **Under which conditions does the considered model admit a free disease and endemic equilibrium points and how can one derive their local and/or global stability in terms of the Basic Reproduction Number?**

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Dynamics of a delayed reaction-diffusion prey-predator model with Hattaf-Yousfi functional response

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Abstract:

The insertion of reaction-diffusion in prey-predator model provide a more realistic representation of how prey-predator interact and disperse in their environment. For that, in this study, we propose a delayed reaction-diffusion prey-predator model with Hattaf-Yousfi functional response. First, we prove that our proposed model is mathematically and ecologically well-posed. Moreover, we investigate whether the stability of equilibria and the condition when the Hopf-bifurcation exist. Finally, to support our theoretical analysis, we also include numerical simulations.

Keywords: Ecology, prey-predator, reaction-diffusion, stability, Hopf-bifurcation, Hattaf-Yousfi functional response.

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The influence of the Hardy-Leray potential on some fractional-order equations

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Abstract:
The main goal of this paper is to prove existence and non-existence results for deterministic Kardar–Parisi–Zhang type equations involving a non-local "gradient term" and a Hardy potential term.

Keywords: fractional elliptic equations; nonlocal gradient term; Hardy potential; stationary Kardar-Parisi-Zhang equations; existence and nonexistence results.

1 Introduction

We will see in this paper how the presence of a Hardy potential affects some results obtained previously, in [3], for a fractional KPZ type problem. More precisely, let $\Omega \subset \mathbb{R}^N$, $N \geq 2$, be a bounded domain with boundary $\partial\Omega$ of class C^2, containing the origin. For $s \in (0,1)$, $\lambda > 0$ and $f \gneqq 0$ in a suitable Lebesgue space, we we deal with the question of existence of solutions to the Kardar–Parisi–Zhang type problem with a non-local "gradient term" $(-\Delta)^{\frac{s}{2}}$ and a Hardy potential :

$$\begin{cases} (-\Delta)^s u = \lambda \dfrac{u}{|x|^{2s}} + |(-\Delta)^{\frac{s}{2}} u|^q + \rho f(x), & \text{in } \Omega, \\ \qquad u = 0, & \text{in } \mathbb{R}^N \setminus \Omega, \end{cases} \tag{KPZ}$$

where $q > 1$ and $\rho > 0$ are real parameters. Note that $\lambda \in (0, \Lambda_{N,s})$ with $\Lambda_{N,s} := 2^{2s} \frac{\Gamma^2(\frac{N+2s}{4})}{\Gamma^2(\frac{N-2s}{4})}$ being the best constant in the fractional Hardy inequality:

$$\frac{a_{N,s}}{2} \int_{\mathbb{R}^N} \int_{\mathbb{R}^N} \frac{|\phi(x) - \phi(y)|^2}{|x-y|^{N+2s}} \, dx \, dy \geq \Lambda_{N,s} \int_{\mathbb{R}^N} \frac{\phi^2}{|x|^{2s}} \, dx, \; \forall \phi \in C_0^\infty(\mathbb{R}^N).$$

Here, we denote by $(-\Delta)^s$ the fractional Laplacian which, for all $u \in C_c^\infty(\mathbb{R}^N)$, is defined by

$$(-\Delta)^s u(x) := a_{N,s} \text{ p.v.} \int_{\mathbb{R}^N} \frac{u(x)-u(y)}{|x-y|^{N+2s}} dy = a_{N,s} \lim_{\epsilon\to 0^+} \int_{\mathbb{R}^N \setminus B_\epsilon(x)} \frac{u(x)-u(y)}{|x-y|^{N+2s}} dy,$$

where $a_{N,s} := 2^{2s} s \pi^{-\frac{N}{2}} \frac{\Gamma(\frac{N}{2}+s)}{\Gamma(1-s)}$ is a normalization constant and Γ denotes the Euler's gamma function.

2 Preliminaries

We give, in this section, a key regularity result for the fractional Poisson problem with a Hardy potential:

$$\begin{cases} (-\Delta)^s u = \lambda \dfrac{u}{|x|^{2s}} + f, & \text{in } \Omega, \\ u = 0, & \text{in } \mathbb{R}^N \setminus \Omega, \end{cases} \tag{P}$$

For $0 < \lambda < \Lambda_{N,s}$, we set

$$\mu(\lambda) = \frac{N-2s}{2} - \alpha_\lambda. \tag{1}$$

where α_λ satisfies

$$\lambda = \lambda(\alpha_\lambda) = \lambda(-\alpha_\lambda) = \frac{2^{2s}\,\Gamma(\frac{N+2s+2\alpha_\lambda}{4})\Gamma(\frac{N+2s-2\alpha_\lambda}{4})}{\Gamma(\frac{N-2s+2\alpha_\lambda}{4})\Gamma(\frac{N-2s-2\alpha_\lambda}{4})}, \tag{2}$$

Let us assume the existence of a positive real number a_0 such that

$$f|x|^{-\mu(\lambda)-a_0} \in L^1(\Omega)$$

and let u be the unique solution of (P). In that case, using the representation formula and some Green's function estimates, we affirm that

$$\left\| (-\Delta)^{\frac{s}{2}} u \right\|_{L^p(\Omega,|x|^{-\mu(\lambda)}dx)} \le C(\Omega,\lambda,a_0) ||f||_{L^1(\Omega,|x|^{-\mu(\lambda)-a_0}dx)} \text{ for all } 1 \le p < \frac{N}{N-s}, \tag{3}$$

3 Existence result

Having established the prior regularity result for problem (P), we can use a fixed point argument to prove our main existence result for (KPZ).

In fact, let $s \in (0,1)$, $0 < \lambda < \Lambda_{N,s}$ and $f \in L^1(\Omega)$ such that

$$f|x|^{-\mu(\lambda)-a_0} \in L^1(\Omega), \quad \text{for some } a_0 > 0.$$

Assume that $1 < q < \overline{q} = \frac{N}{N-s}$. Then problem (KPZ) has a weak solution for ρ small enough.

4 Non existence results

- In comparison with the basic case $\lambda = 0$ treated in [3], we can't prove the existence of a solution to (KPZ) for every $q \in (1, \infty)$ even if we assume f to be highly regular. In point of fact, we show, in this section, the existence of a critical exponent q_+ depending only on λ and s, such that problem (KPZ) has no weak solution for $q > q_+$.
- Furthermore, for the appropriate data, we validate that (KPZ) has also no weak solution for ρ large enough.

Our proofs are based on a careful selection of test funtions.

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Existence of Solutions for a Class of Fractional Kirchhoff-type Systems in $\mathbb{R}^N$ with Non-standard Growth

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Abstract:
This talk is concerned with the existence and multiplicity of nontrivial solutions for a class of Kirchhof-type systems in $\mathbb{R}^N$ involving the fractional pseudo-differential operators defined as the generalizations of the $p(x)$-Laplace operator. Our main tools come from a direct variatioal methods, the Mountain Pass Theorem, the symmetric Mountain Pass Theorem and the Fountain Theorem in critical point theory. The obtained results of this note significantly contribute to the study of Kirchhoff-type systems in the sense that our situation covers not only differential operators of fractional order but also nonhomogeneous differential operators in Sobolev spaces with variable exponent.

Keywords: Kirchhof type systems, Fractional $p(x, .)$-Laplace operators, Generalized fractional Sobolev spaces, Variational methods.

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Existence Results For A Fractional $(p(x,.),q(x,.))$-Kirchhoff Type Elliptic System

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Abstract:
This talk is concerned with the existence and the multiplicity of weak solutions for a nonlocal fractional elliptic system of $(p(x,.),q(x,.))$ Kirchhoff type with weight and homogeneous Dirichlet boundary conditions. The approach is based on the three critical points theorem introduced by Recceri and on the theory of general fractional Sobolev spaces with variable exponents

Keywords: Elliptic systems, Generalized fractional Sobolev spaces, Weighted variable exponent spaces, Three critical-points theorem.

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Variational methods for a fractional $p(x,.)$-bi-nonlocal problem of elliptic type

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Abstract:
In this work, by using the Mountain pas theorem and Elknad's variational principle, we prove the existence of weak solutions for a class of Kirchhoff type problems involving the fractional p(x)-Laplacian problem and contain a bi-nonlocal term. The main goal is to treat a class of Kirchhoff functions and apply the appropriate variational method based on the conditions of each example function.

Keywords: General nonlocal integro-differential equation, variational methods, $p(x)$-Kirchhoff type problem..

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Lipschitz functions class for the generalized Dunkl transform

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Abstract:

This paper is intented to establish the analogue of Titchmarsh's theorem for the Dunkl generalized transform on the real line.

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Communication Info

Titre: second Hankel-clifford

Abstract

In this work, we generalize theorem of Hardy for the second Hankel-Clifford transform .f on (0, +∞), such that $\|f\|_{L^P_\mu} = \int_0^\infty |f(x)|^P x^\mu dx < \infty$.

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The Cooperative Lane-Emden System With Hardy Potential

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Abstract:
The main goal of our work is to show the existence and non-existence of positive solution to the following cooperative Lane -Emden system with singular Hardy potential:

$$(GS)\begin{cases}(-\Delta)u = \lambda_1 \dfrac{v}{|x|^2} + f(x, v, \nabla v) \quad in\ \Omega,\\[2ex] (-\Delta)v = \lambda_2 \dfrac{u}{|x|^2} + g(x, u, \nabla u) \quad in\ \Omega,\\[2ex] u = v = 0 \quad on \quad \partial\Omega\end{cases}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded domain containing the origin and $N \geq 3$.

Since we are looking for positive solutions, it is natural to require that f and g are non-negative functions.

In particular we will focus our analysis in the cases:

$$f = v^p, g = u^q, \quad f = |\nabla v|^p, g = |\nabla u|^q, \tag{0.1}$$

Keywords:Singular elliptic system, Hardy inequality, Picone identity.

1 Introduction

The main goal of this work is to analyze the question of existence and non existence of nonnegative solutions for the cooperative singular system given by

$$(GS) \quad \begin{cases} -\Delta u &= \lambda_1 \dfrac{v}{|x|^2} + f(x, v, \nabla v) \quad \text{in } \Omega,\\ -\Delta v &= \lambda_2 \dfrac{u}{|x|^2} + g(x, u, \nabla u) \quad \text{in } \Omega,\\ u = v &= 0 \qquad\qquad\qquad \text{on} \quad \partial\Omega \end{cases}$$

where $\lambda_1, \lambda_2 > 0$.

We will consider two main cases:

1. the case where $f(x,v) = v^p$ and $g(x,u) = u^q$, using suitable argument, we show the existence of a critical curves in $H(p,q)$ such that the system (S) has a solution according to the sign of H.

2. the case $f(x,v) = |\nabla v|^p$and $g(x,u) = |\nabla u|^q$. Here the situation is more complicated and we need to use a suitable weighted Sobolev inequality in order to get the critical curves of existence.

2 Literature Review

In the case of one equation, the previous system is reduced to the equation

$$-\Delta u = \lambda \frac{u}{|x|^2} + f(x,u,\nabla u) \text{ in } \Omega, u \geq 0 \text{ in } \Omega. \tag{2.1}$$

It is well known that problem (2.1) has a solution if $\lambda \leq \Lambda_N$ under additional hypotheses on the nonlinear term $f(x,u,\nabla u)$.
Setting $w_\pm = |x|^{-\frac{N-2s}{2} \pm \sqrt{\Lambda_N - \lambda}}$, then $w_\pm$ solves the equation

$$-\Delta w = \lambda \frac{w}{|x|^2} \text{ in } (\mathbb{R}^N \setminus \{0\}), \tag{2.2}$$

In order to obtain the existence and non existence of nonnegative solution to problem (2.1), we need to estimate the behavior of u near the singular point 0, combining with the form of the non linear term $f(x,u,\nabla u)$.

Notice that $u \geq 0$ in Ω satisfies

$$-\Delta u \geq \lambda \frac{u}{|x|^2} \text{ in } B_r(0),$$

then $u(x) \geq C|x|^{-\mu(\lambda)}$ in $B_r(0)$ with $\mu(\lambda) = \dfrac{N-2s}{2} - \sqrt{\Lambda_N - \lambda}$.

• if $f \in L^1(\Omega)$ does not depends on u and ∇u ($f(x,u,\nabla u) := f(x)$), then problem (2.1) has a solution if and only if $\displaystyle\int_\Omega |x|^{-\mu(\lambda)} f(x)\, dx < \infty$ where $\mu(\lambda)$ is defined above.

• If $f(x,u) = u^p$, then existence of a nonnegative supersolution holds if and only if $p < p_+(\lambda) = \frac{\mu(\lambda)+2s}{\mu(\lambda)}$.

We refer for instance [6] for more details.

• If $f(x,u,\nabla u) = |\nabla u|^q + \alpha h$, then the authors in [3] proved that the existence of a solution if and only if $\lambda \leq \Lambda_N$ and $q < q_+(\lambda) = \frac{\mu(\lambda)+2}{\mu(\lambda)+1} < 2$.

In the case of system with Hardy potential: the authors in [5] consider the system

$$\begin{cases} -\Delta u = \lambda_1 \dfrac{u}{|x|^2} + v^p \quad \text{in } \Omega, \\ -\Delta v = \lambda_2 \dfrac{v}{|x|^2} + u^q \quad \text{in } \Omega, \\ u, v > 0 \quad \text{in } \Omega. \end{cases} \tag{2.3}$$

Using an iteration process, they get existence and non existence of a nonnegative solution.

3 Another section

the main difficulty to study System (GS) is the fact that non estimate on the behavior of u, v near the origin can be obtained.

Hence we begin by proving the next key lemma.

Lemma 3.1. *Assume that* $\sqrt{\lambda_1\lambda_2} \le \Lambda_N$ *and let* f, g *be nonnegative function. Let* (u, v) *be a weak positive supersolution solution to System* (LS)*. Then*

$$u + v \ge C|x|^{-\mu(\sqrt{\lambda_1\lambda_2})} \text{ in } B_r(0). \tag{3.1}$$

In addition we have

$$\int_\Omega (f+g)|x|^{-\mu(\sqrt{\lambda_1\lambda_2})}dx < \infty.$$

4 Results and Discussion

Our main non-existence results for the *potential case* is given by the next theorem:

Theorem 4.1. *Suppose that* $\min\{p, q\} \ge \frac{N-\mu(\sqrt{\lambda_1\lambda_2})}{\mu(\sqrt{\lambda_1\lambda_2})}$, *then System* (PS) *has non weak supersolution.*

Theorem 4.2. *Let* $\lambda_1, \lambda_2 > 0$ *be such that* $\sqrt{\lambda_1\lambda_2} \le \Lambda_N$.

1. $\frac{\mu(\sqrt{\lambda_1\lambda_2})+2}{\mu(\sqrt{\lambda_1\lambda_2})} < p < \frac{N-\mu(\sqrt{\lambda_1\lambda_2})}{\mu(\sqrt{\lambda_1\lambda_2})}$, $q \le p$ *and* $\mu(\sqrt{\lambda_1\lambda_2})\frac{pq-1}{p+1} > 2$,
   Or

2. $\frac{\mu(\sqrt{\lambda_1\lambda_2})+2}{\mu(\sqrt{\lambda_1\lambda_2})} < q < \frac{N-\mu(\sqrt{\lambda_1\lambda_2})}{\mu(\sqrt{\lambda_1\lambda_2})}$, $p \le q$ *and* $\mu(\sqrt{\lambda_1\lambda_2})\frac{pq-1}{q+1} > 2$,

then System (PS) *has non weak positive supersolution.*

For the existence, we have:

Theorem 4.3. *Let Ω be a bounded domain containing the origin and that $\sqrt{\lambda_1\lambda_2} \leq \Lambda_N$.*

1. *Suppose that $p, q < \frac{\mu(\sqrt{\lambda_1\lambda_2})+2}{\mu(\sqrt{\lambda_1\lambda_2})}$, namely $(p, q) \in A$. Then System (PS) has a positive supersolution.*

For *the gradient case*, using an iteration technique, we get:

Theorem 4.4. *Suppose that $\min\{p, q\} \geq \frac{N-\mu(\sqrt{\lambda_1\lambda_2})}{\mu(\sqrt{\lambda_1\lambda_2})+1}$, then System (PS) has non weak supersolution.*

Theorem 4.5. *Let $\Omega \subset (R)^N$ be a bounded domain such that $0 \in \Omega$ and $N \geq 3$. Assume that $0 < \lambda_1, \lambda_2 \leq \Lambda_N$ and suppose that $p, q > 0$ satisfying the following assumptions:*

1. $q < \frac{\mu(\sqrt{\lambda_1\lambda_2})+2}{\mu(\sqrt{\lambda_1\lambda_2})+1} < p < \frac{N-\mu(\sqrt{\lambda_1\lambda_2})}{\mu(\sqrt{\lambda_1\lambda_2})+1}, q \leq p$ *and* $H(p, q) < 0$,

2. $p < \frac{\mu(\sqrt{\lambda_1\lambda_2})+2}{\mu(\sqrt{\lambda_1\lambda_2})+1} < q < \frac{N-\mu(\sqrt{\lambda_1\lambda_2})}{\mu(\sqrt{\lambda_1\lambda_2})+1}$, $p \leq q$ *and* $H(p, q) < 0$,

Then the System (QS) has no nonnegative weak solution

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Exact Controllability of the 1-D Beam Equation with Piezoelectric Actuator

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Abstract:
We consider initial and boundary value problems modelling the vibrations of a beam with piezoelectric actuator. The simplest model leads to the Bernoulli-Euler beam equation with right hand side given by a Dirac mass multiplied by a real valued time function, representing the voltage applied to the actuator. The aim of this paper is to study the exact controllability problem using the optimal control approach. Finally, we illustrate these results with numerical simulations.

Keywords: Beam model, exact controllability, optimal control, piezoelectric actuator.

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Mathematical modeling of HIV transmission in a heterosexual population: incorporating memory conservation

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Abstract:
HIV disease is a major global public health concern since its appearance in the early 1980s. Many mathematical models have been conducted to understand, control, and predict its spread. In this paper we propose a mathematical model with memory efect modelling HIV transmission in a heterosexual population divided into two age classes; young-class (15–24 years old) and grown-class (25 years old and over). The goal of dividing the population according to age is to identify the most vulnerable class to the virus based on their sexual activity and make accurate predictions about HIV transmission. First, we determine the biologically signifcant space for the study, and we prove the existence of a unique solution. Then we divide the principal model into four sub-models: young-people, grown-people, young-men linked to grown-women, grown-men linked to young-women. The basic reproduction number associated to each sub-model is derived. According to the four sub-models, we have found that, if the basic reproduction number is below unity, then the free disease equilibrium state is locally asymptotically stable. Numerical simulations are provided to validate the theoretical results and discuss the local stability of the endemic equilibrium states of each sub-model. We conclude that incorporating memory conservation gives more realistic results, where reaching a stable state takes higher time. As well, memory efect can play the role of prior knowledge about the disease and experience accumulated over years.

Keywords: HIV · Heterosexual population · Caputo derivative · Fractional differential system.

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International Conference on Recent Advances in Partial Differential Equations and Applications (ICRAPDEA'23)

July 27-28, 2023, Faculty of Sciences Casablanca, Hassan II University, Casablanca Morocco

On the equivalence of K-Functionals and modulus of Smoothness Constructed by the generalized Fourier-Bessel transform

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Abstract :
Using a generalized translation operator, we define generalized modulus of smoothness in the space $\mathrm{L}^2_{\alpha,n}$. Based on the generalized Bessel operator we define Sobolev-type space and K-functionals. The main result of this paper is the proof of the equivalence theorem for a K-functional and a modulus of smoothness for the generalized Fourier-Bessel transform.

Keywords: generalized Fourier-Bessel transform; generalized translation operator; K-functional.

Mathematics Subject Classification: 42B10.

1 Introduction

The K-functional is a useful tool in several areas of mathematics, e.g. functional analysis, harmonic analysis and the theory of ordinary differential equations. The equivalence between the K-functional and the modulus of smoothness has been studied by many authors (see [6] for functions defined on the n-dimensional Euclidean space $\mathbb{R}^n$ and [7] Chapter 6 for the univariate case). In [8], we proved this equivalence between the K-functional and the modulus of smoothness for the Dunkl transform on $\mathbb{R}^d$, using a generalized spherical mean operator.

In this paper, we prove an analog of this result in the Hilbert space $\mathrm{L}^2_{\alpha,n}$. For this purpose, we use a generalized translation operator in the place of a generalized spherical mean operator.

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International Conference on Recent Advances in Partial Differential Equations and Applications (ICRAPDEA'23)

July 27-28, 2023, Faculty of Sciences Casablanca, Hassan II University, Casablanca Morocco

A class of fractional parabolic reaction-diffusion systems with control of total mass: theory and numerics

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Abstract:
In this talk, we present results on the global-in-time existence of strong solutions to a class of fractional parabolic reaction-diffusion systems posed in a bounded open subset of $\mathbb{R}^N$. The nonlinear reactive terms are assumed to satisfy natural structure conditions which provide nonnegativity of the solutions and uniform control of the total mass. The diffusion operators are the fractional Laplacian. Global existence of strong solutions is proved under the assumption that the nonlinearities are at most of polynomial growth. Our results extend previous results obtained when the diffusion operators are the classical Laplacian. On the other hand, we use numerical simulations to examine the global existence of solutions to systems with exponentially growing right-hand sides, which remains so far an open theoretical question even in the classical case.
Keywords: Reaction-diffusion system, fractional diffusion, strong solution, global existence, numerical simulation.

1 Introduction

The purpose of this work is the study of global existence in time of nonnegative strong solutions to fractional parabolic reaction-diffusion systems of the form:

$$(S)\qquad \begin{cases} \forall i = 1, \ldots, m, & & & \\ \partial_t u_i + d_i(-\Delta)^s u_i &=& f_i(u_1, \ldots, u_m), & \text{in} \quad (0,T)\times\Omega, \\ u_i(t,\mathbf{x}) &=& 0, & \text{in} \quad (0,T)\times(\mathbb{R}^N\setminus\Omega), \\ u_i(0,\mathbf{x}) &=& u_{0i}(\mathbf{x}), & \text{in} \quad \Omega, \end{cases}$$

where Ω is a bounded regular open subset of $\mathbb{R}^N$, $T > 0$, $0 < s < 1$, $m \in \mathbb{N}^*$ and for each $i = 1, \ldots m$, the diffusion coefficient $d_i > 0$ and f_i is locally Lipschitz continuous. $(-\Delta)^s$ denotes the fractional Laplacian defined by

$$(1)\qquad (-\Delta)^s\phi(\mathbf{x}) := a_{N,s}\ \text{P.V.} \int_{\mathbb{R}^N} \frac{\phi(\mathbf{x}) - \phi(\mathbf{y})}{\|\mathbf{x}-\mathbf{y}\|^{N+2s}}\, d\mathbf{y},$$

where $a_{N,s} := \frac{s2^{2s}\Gamma\left(\frac{N+2s}{2}\right)}{\pi^{\frac{N}{2}}\Gamma(1-s)}$, P.V. stands for the the Cauchy principal value, and $\|\cdot\|$ is the Euclidean norm of $\mathbb{R}^N$. For more details about this operator, see, for instance, [8] and references therein.

Interest in this type of models has grown recently, particularly for applications in biology, ecology, environment science and population dynamics.

As we are only interested in nonnegative solutions, as in applications the unknown $\mathbf{u} = (u_1, \cdots, u_m)$ represents for example concentrations of chemical species or population densities. Therefore, the initial data have to be chosen nonnegative ; $u_{0i} \geq 0$ for each $i \in \{1, \cdots, m\}$.

It is also well-known that the solutions (as long as they exist) remain nonnegative, provided that the reaction terms f_i satisfy the so-called "quasi-postivity" property, namely:

$$(\mathbf{P}) \qquad \forall 1 \leq i \leq m, \; f_i(r_1, \cdots, r_{i-1}, 0, r_{i+1}, \cdots, r_m) \geq 0, \; \forall \mathbf{r} = (r_1, \cdots, r_m) \in [0, +\infty)^m.$$

Furthermore, the control of total mass is fulfilled if

$$(\mathbf{M}) \qquad \exists (a_1, \cdots a_m) \in (0, +\infty)^m \text{ such that } \sum_{i=1}^{m} a_i f_i \leq 0.$$

Let us emphasize that properties **(P)** and **(M)** are satisfied in many applications, *e.g.* in models describing evolution phenomena involving both spatial diffusion and chemical reactions.

2 Literature Review

In order to put our work in context and highlight the novelty of our work, let us briefly review the existing literature.

• Case $s = 1$. In addition to the structure **(P)**+ **(M)**, some growth restrictions and extra structure on the nonlinearities are needed if one expects global existence of strong solutions. This issue has been intensively studied, especially when the initial data are bounded. Let us review some sufficient conditions on the f_i's guaranteeing the global existence of a *strong* solution:

(i) *triangular structure* (see (2)) and polynomial growth. For more details, we refer to [12] and references therein;

(ii) when the growth of the f_i's is slightly stronger than polynomial, few results are known, and only for $m = 2$. The most-studied model by far is $f_2(u_1, u_2) = -f_1(u_1, u_2) = u_1 e^{u_2^\beta}$. In this case, the existence of strong solutions is proved in [10] for $\beta < 1$, and [6] for $\beta = 1$. However, the global existence is established under a restrictive assumption on the size of u_{01}. On the other hand, the problem is still open for $\beta > 1$.

• Case $0 < s < 1$. Unlike the case $s = 1$, relatively little is known. To our knowledge, the existing works fall into two broad categories: either the domain Ω is bounded and the right-hand sides are of potential-gradient (see [5]) or gradient-gradient (see [4]) type; or $\Omega = \mathbb{R}^N$ and the r.h.s. are of polynomial or exponential growth, see [1, 2].

3 Results and Discussion

Our talk is divided into two parts.

• Theoretical part. We extend two known results in the classical case ([12, Theorem 3.5] and [11, Theorems 1 and 2]) to the fractional case. Our first main result reads as:

Theorem 1 ([9, Theorem 4.2]). *Let $(u_{01}, \dots, u_{0m}) \in (L^\infty(\Omega)^+)^m$ where $m > 2$. Besides* **(P)***, assume that f_i is at most polynomial for each $i = 1, \dots, m$. Moreover, assume that there exist a vector $\mathbf{b} \in [0, +\infty)^m$, and a lower triangular invertible matrix $Q \in \mathcal{M}_m([0, +\infty))$, such that*

$$\forall \mathbf{r} = (r_1, \dots, r_m) \in [0, +\infty)^m, \; Q\mathbf{f}(\mathbf{r}) \le \Big[1 + \sum_{i=1}^{m} r_i\Big]\mathbf{b}. \tag{2}$$

Then, System (S) admits a unique nonnegative global strong solution.

Now, let us consider a typical case of reversible chemical reactions for three species, namely

$$\begin{cases} \partial_t u_1 + d_1(-\Delta)^s u_1 &= f_1(u_1, u_2, u_3) & \text{in } (0,T)\times\Omega, \\ \partial_t u_2 + d_2(-\Delta)^s u_2 &= f_2(u_1, u_2, u_3) & \text{in } (0,T)\times\Omega, \\ \partial_t u_3 + d_3(-\Delta)^s u_3 &= f_3(u_1, u_2, u_3)) & \text{in } (0,T)\times\Omega, \\ u_1 = u_2 = u_3 &= 0, & \text{in } (0,T)\times(\mathbb{R}^N\setminus\Omega), \\ u_1(0,\mathbf{x}) &= u_{01}(\mathbf{x}), & \text{in } \Omega, \\ u_2(0,\mathbf{x}) &= u_{02}(\mathbf{x}), & \text{in } \Omega, \\ u_3(0,\mathbf{x}) &= u_{03}(\mathbf{x}), & \text{in } \Omega, \end{cases} \tag{3}$$

where

$$f_1 = \alpha_1 g, \; f_2 = \alpha_2 g, \; f_3 = -\alpha_3 g \text{ with } g = u_3^{\alpha_3} - u_1^{\alpha_1} u_2^{\alpha_2} \text{ and } \alpha_1, \alpha_2, \alpha_3 \ge 1.$$

This system naturally arises in chemical kinetics. Besides, it fulfills the properties **(P)** and **(M)**.

Theorem 2 ([9, Theorem 4.1]). *Assume that $(u_{01}, u_{02}, u_{03}) \in (L^\infty(\Omega)^+)^3$. System* (3) *admits a unique nonnegative global strong solution in the following cases:*

(i) $\alpha_1 + \alpha_2 < \alpha_3$ *;*

(ii) $\alpha_3 = 1$ *whatever are α_1 and α_2 ;*

(iii) $d_1 = d_3$ *or $d_2 = d_3$ whatever are α_1,α_2 and α_3 ;*

(iv) $d_1 = d_2 \neq d_3$ *for any $(\alpha_1, \alpha_2, \alpha_3) \in [1, +\infty)^2 \times (1, +\infty)$ such that $\alpha_1 + \alpha_2 \neq \alpha_3$.*

• Numerical part. we present some numerical simulations to address the open theoretical question (even in the case $s = 1$) about the global existence of solutions to System (S) in the case where $m = 2$, $f_2(u_1, u_2) = -f_1(u_1, u_2) = u_1 e^{u_2^\beta}$ with $\beta > 1$. It should be noted that the proposed method was already applied with success in [4, 3].
Our numerical experiments have showed that our System has nonnegative solutions that can be computed for a large final time, see [9, 7].

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International Conference on Recent Advances in Partial Differential Equations and Applications (ICRAPDEA'23)

July 27-28, 2023, Faculty of Sciences Casablanca, Hassan II University, Casablanca Morocco

Population Dynamics Under Climate Change: Influences of Allee Effect and Variable Speed

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Abstract:

The Allee effect was first described in 1931 by Warder Clyde Allee (1885-1955). It occurs in some species and only for low densities and it is characterized by a positive correlation between the density of a population and its growth rate. There are several studies concerning population dynamics under climate change, among which we can cite the study of the persistence of a population with logistic growth in the face of climate change of constant speed [1], or variable speed which depends on time [2]. In addition, a recent study [3] provides a complete picture of the persistence of species in a prey-predator system. The study of population dynamics under climate change in the presence of an Allee effect is less studied in the literature, see for instance [4] with a numerical study in 2 dimensions.

We are interested in population persistence under the Allee effect in a diffusive environment with climate change that is modeled by a spatio-temporal heterogeneity as a function of a moving variable. So, could a population under the Allee effect persist in the face of climate change of varying speed? To answer this question, we will address the persistence or extinction of the species depending on the strength of the Allee effect and the nature of the speed of climate change. The model we consider here is derived from Fisher's classical model of population dynamics [4]. We will use this dynamic equation to study the persistence of the species. In the case where the speed of climate change is constant, we will transform our model using a change of variable to another that is equivalent, which will allow us to draw the phase portrait of the population and monitor its evolution within the climate envelope. In the case where the speed of climate change is variable, we will content ourselves with simulations and numerical analysis to draw general results on the persistence of the species.

Keywords: Strong Allee effect, Weak Allee effect, Climate change, Climate envelope, Reaction-diffusion equation, Persistence, Extinction.

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International Conference on Recent Advances in Partial Differential Equations and Applications (ICRAPDEA'23)

July 27-28, 2023, Faculty of Sciences Casablanca, Hassan II University, Casablanca Morocco

Accelerated Residual New Iterative Method for Solving generalized Burgers-Huxley Equation

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Abstract:
Nonlinear partial differential equations are essential in many fields, such as physics, chemistry, biology, mathematics, and engineering. In recent years, several semi-analytical methods have been proposed in the literature such as the Adomian Decomposition Method (ADM) and the New Iterative Method (NIM).
In the present work, we present a new formulation of NIM and introduce a parameter so-called as convergence control parameter. The proposed method is easy to use and implement. The convergence study will be specified. We provide in this paper a study to approve that the new formulation effectively improves the classical NIM method and to analyze the convergence as a function of the control parameter. The best suitable value of this parameter is determined in the global quadratic residual approximation to ensure the fastest convergence of the iterative scheme.
We will apply this new method to solve the generalized Burgers-Huxley equation and we will give the convergence conditions. Then, a comparison with classical methods as well as the exact solution will be presented.
Depending on the control parameter, the proposed methods have the advantage of rapid convergence and easy implementation. The numerical results will be presented to validate the theoretical part and are satisfactory.

Keywords: Adomian Decomposition Method, New Iterative Method, Rsidual control parameter, Vonvergence Analysis.

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PDEs & Applications

International Conference on Recent Advances in Partial Differential Equations and Applications (ICRAPDEA'23)

July 27-28, 2023, Faculty of Sciences Casablanca, Hassan II University, Casablanca Morocco

Optimal control for Dirac bilinear systems

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Abstract:
This article addresses the optimal control problem for Dirac bilinear systems, namely there who bounded from some space V to its dual V^*. We consider two cases: optimal control in a finite time interval with application to endpoint problem and the scenario of an infinite time interval with application to stabilisation problems.

Keywords: Optimal control, stabilization, Variational method.

1 Introduction

This paper focuses on investigating the optimal control problem for a bilinear Dirac systems. Bilinear Dirac systems play a significant role in various areas of physics and mathematics.
The study of bilinear Dirac systems poses significant mathematical challenges. The presence of Dirac delta functions introduces non-smoothness and nonlinearity, requiring specialized techniques to analyze and solve these systems. The understanding of their properties and behavior contributes to advancing our knowledge of quantum mechanics, mathematical physics, and nonlinear systems theory.
Our objective is to minimize a quadratic cost function within a set of admissible controls. The definition of the optimal control problem is as follows:

$$\min_{v\in U_{ad}} J(v) := \left(\|u(T) - u_1\|^2 + \int_0^T \|u(t)\|^2 dt + 1 \int_0^T |v(t)|^2 dt \right), \tag{1}$$

This minimization is subject to the dynamical system described by:

$$\begin{cases} \dot{u}(t) = \Delta u(t) - v(t)u(t)\delta_\gamma + v(t)f, & (0,T)\times\Omega \\ u = 0, & (0,T)\times\partial\Omega \\ u(0) = u_0 \in L^2(\Omega), \end{cases} \tag{2}$$

Ω and ω are an open and bounded domain of $\mathbb{R}^n$ such that $\bar{\omega} \subset \Omega$, γ the boundary of $\bar{\omega}$ and $f \in (H_0^1(\Omega))^*$. In the above equations, $0 < T \leq +\infty$, $U_{ad} = \{v \in L^\infty(0,T); \quad \forall t \in (0,T),\ 0 \leq v(t) \leq M\}$, v denotes the control input and u the corresponding state and a, b are positive weight coefficients, they determine the relative importance and weighting of the state and control variables in the cost function, . Here, $\langle \cdot, \cdot \rangle$, $\| \cdot \|$ refer to the scalar product and corresponding norm in $L^2(\Omega)$.

Throughout the article, we assume that the following four assumptions hold:

- the linear continuous operator $B : H_0^1(\Omega) \to H_0^1(\Omega)^*, Bu = u\delta_\gamma$ is defined by $\langle Bu_1, u_2 \rangle = \int_\gamma u_1 u_2 ds$.
- $v(t)$ is a real-valued control such that $v \in U_{ad} = \{v \in L^\infty(0,T) \mid 0 \leq v(t) \leq M\}$, $M \geq 0$.

2 Literature Review

Previous research has explored cases involving bounded control operators B and expressed the necessary conditions that generally allow the optimal control to be written as a time-varying feedback control (see [2, 3]). Additionally, some works have studded the optimal control problem for unbounded specific systems. In a similar vein, Breiten et al. [1] investigated the problem of unconstrained endpoint optimal control for a class of multiplicative control systems involving a control operator $B : V \mapsto X$. While this model traditionally applies to the Fokker-Planck equation. However, in this study, we extend our investigation to Dirac control operator witch provides a mathematical representation for concentrated effects or point source.

3 Results and Discussion

This paper is devoted to the solution of the problem (1). We first study the formulation of necessary conditions subject of the following theorem. Then, consider the case of of constraints on the final state, and finally analysie the case of the infinite time horizon and stability by an optimal control.

3.1 Finite time interval

Theorem 1.
Given $T > 0$, the optimization problem (1) has a feasible solution v^* that satisfies the following relation:

$$v^*(t) = \min(M; \max(0; \frac{1}{2}\langle p(t), u^*(t)\delta_\gamma + f\rangle_{H_0^1(\Omega)^*, H_0^1(\Omega)} \tag{3}$$

where u^* is the weak solution to the system (2) *that corresponds to v^* and p is the weak solution to the following system*

$$\begin{cases} \dot{p}(t) = -\Delta p(t) + v^*(t)B^*p(t) - 2u^*(t) \\ p(T) = 2(u^*(T) - u_1)\cdot \end{cases} \tag{4}$$

In the upcoming theorem, our objective is to solve an optimal control problem with an endpoint constraint. To accomplish this, we will solve an optimal control problem with an endpoint constraint, as described below

$$\min_{v \in U_c} J_c(v) := \min_{v \in U_c} a \int_0^T \|u(t)\|^2 dt + b \int_0^T |v(t)|^2 dt \tag{5}$$

where $a, b \geq 0$, $U_c = \{v \in U_{ad} \mid u(T) = u_d\}$ and $u_d \in \{x \in X \mid \exists v \in U_{ad}, u_v(T) = x\}\cdot$ We let us consider a decreasing sequence ϵ_n that converges to 0, and let v_n be the corresponding solution to problem (1) and where a is replaced by $\epsilon_n a$ and b by $b\epsilon_n$. The cost function associated with this problem is denoted as (J_{ϵ_n}). Let u_n denote the solution of the system (2) corresponding to v_n. Leveraging the results obtained from the previous theorem, we address problem (5) in the following corollary:

Corollary 2.
The problem (5) possesses a solution v^*. Additionally, any weak limit of the sequence (v_n) solutions of the problem (1) is also a solution of (5)*.*

3.2 Infinite time interval

Upto now we have always assumed that $T < +\infty$. We shall now see how the optimal control depend on T and study their behaviour as $T \to +\infty$. With the same notation as in finite time horizon, consider the minimization without constraints of the quadratic cost function J

$$J(v) = a \int_0^{+\infty} \|u(t)\|^2 dt + b \int_0^{+\infty} |v(t)|^2 dt, \tag{6}$$

where a and b are non-negative constants, $v \in U$, and u is the corresponding mild solution of the system (2). Here, we take $U = \{v \in L^\infty(0, +\infty) \mid 0 \leq v(t) \leq M\}$, with M being a positive constant. The optimal control problem is then formulated as follows:

$$\begin{cases} \min J(v) \\ v \in U_{ad} = \{v \in U \ \ / \ J(v) < +\infty\} \end{cases} \tag{7}$$

Assuming that $U_{ad} \neq \emptyset$, our objective in this following theorem is to provide a solution to the problem (7). To achieve this, we consider an increasing sequence T_n of time intervals tending to infinity. Then for each T_n, we denote by J_n the cost function of the problem (1) and by (v_n) th optimal which is solution. Let u_n, p_n be respective corresponding solution of (2) and (4).

We are ready to establish the following stabilization result.

Theorem 3. *The problem* (7) *possess a solution* v^**, which is given by the solution of the following system of equations:*

$$\begin{cases} v^*(t) = \min(M; \max(0; \dfrac{1}{2b}\langle p(t), Bu^*(t) + f\rangle_{H_0^1(\Omega)^*, H_0^1(\Omega)})) \\ \dot{p}(t) = -\Delta p(t) + v^*(t)B^*p(t) - 2au^*(t) \\ \dot{u^*}(t) = \Delta u^*(t) - v^*(t)Bu^*(t) - v^*(t)f \\ \lim_{t \to +\infty} \|u^*(t)\| = 0\cdot \end{cases} \tag{8}$$

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K-functional Related to The Deformed Hankel Transform

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Abstract:
The main result of this talk is the proof of the equivalence theorem for a K-functional and a modulus of smoothness for the Deformed Hankel Transform. Before that, we introduce the K-functional associated to the Deformed Hankel Transform.

Keywords: Deformed Hankel transform, generalized translation operator, Jackson's direct theorems, K-Functionals, Modulus of smoothness.

1 Introduction

In [2], Belkina and Platonov established the equivalence theorem for a K-functional and a modulus of smoothness for the Dunkl transform in the Hilbert space $L^2(\mathbb{R}, |x|^{2\alpha+1})$, $\alpha \geq \frac{-1}{2}$ using a Dunkl translation operator. In this talk, we prove the generalization of this theorem for the Deformed Hankel transform $\mathcal{F}_k$, with a parameter $k > \frac{1}{4}$. For this purpose, we use the deformed Hankel translation operator, this result is analogous of the statement proved in ([1], [2], [8], [10], [11]).
We recapitulate some facts about harmonic analysis related to the deformed Hankel transform, consider the differential operator

$$\mathbb{L}_k = |.|\, \lambda_k(.)$$

where Λ_k is the Dunkl Laplacian defined by $\Lambda_k = \frac{d^2}{dx^2} + \frac{2k}{x}\frac{d}{dx} - \frac{k}{x^2}(1-S)$, where $Sf(x) = f(-x)$.
The deformed Hankel kernel $B_k(\lambda x)$ is given,for $k > \frac{1}{4}$, by

$$B_k(\lambda x) = j_{2k-1}(2\sqrt{|\lambda x|}) - \frac{\lambda x}{2k(2k+1)} j_{2k+1}(2\sqrt{|\lambda x|}),$$

where j_α denotes the normalized Bessel function of order α, defined by

$$j_\alpha(u) = 2^\alpha \Gamma(\alpha+1) u^{-\alpha} J_\alpha(u) = \Gamma(\alpha+1) \sum_{m=0}^{\infty} \frac{(-1)^m}{m!\Gamma(\alpha+m+1)} \left(\frac{u}{2}\right)^{2m}.$$

It satisfies the following differential-difference equation

$$|x|\Lambda_k B_k(\lambda x) = -|\lambda| B_k(\lambda x).$$

One can easily find that B_k has the properties

$$B_k(0) = \text{ and } |B_k(\lambda x)| \leq 1 \text{ for all } \lambda, x \in \mathbb{R}$$

and from [9], we have

$$\lim_{\lambda x \to +\infty} B_k(\lambda x) = 0$$

The deformed Hankel translation operator T_y^k is defined by

$$T_y^k f(x) = \int_{\mathbb{R}} f(z) K_k(x,y,z) d\mu_k(z)$$

where $d\mu_k(x) = \frac{1}{2\gamma(2k)} |x|^{2k-1} dx$ and for all $x, y \in \mathbb{R}^*$, the kernel K_k is given by

$$K_k(x,y,z) = 2\Gamma(2k) W_{2k-1}(\sqrt{|x|}, \sqrt{|y|}, \sqrt{|z|}) \nabla_k(x,y,z),$$

where W_α is the positive Bessel kernel given by

$$W_\alpha(u,\, v,\, w)$$

$$= \frac{\Gamma(\alpha+1)}{2^{2\alpha-1}\Gamma(\alpha+\frac{1}{2})\Gamma(\frac{1}{2})} \frac{\left\{[(u+v)^2 - w^2][w^2 - (u-v)^2]\right\}^{\alpha-\frac{1}{2}}}{(uvw)^{2\alpha}} \chi_{]|u-v|,u+v[}(w),$$

and

$$\begin{aligned}
\nabla_k(x,y,z) =& \frac{1}{4}\Big\{1 + \frac{\text{sgn}(xy)}{4k-1}[4k\triangle(|x|,|y|,|z|)^2 - 1] \\
&+ \frac{1}{4}\Big\{1 + \frac{\text{sgn}(xz)}{4k-1}[4k\triangle(|z|,|x|,|y|)^2 - 1] \\
&+ \frac{1}{4}\Big\{1 + \frac{\text{sgn}(yz)}{4k-1}[4k\triangle(|z|,|y|,|x|)^2 - 1]
\end{aligned}$$

and $\Delta(u,v,w) = \frac{1}{2\sqrt{uv}}\big(u+v-w\big)$, $u,v,w \in \mathbb{R}_+^*$. The deformed Hankel transform $\mathcal{F}_k$ is defined by

$$\mathcal{F}_k f(\lambda) = \int_{\mathbb{R}} f(x) B_k(\lambda x) d\mu_k(x), \qquad \lambda \in \mathbb{R}.$$

We have $\mathcal{F}_k(f) \in \mathcal{C}_0(\mathbb{R})$. Moreover we have

$$\left|\left|\mathcal{F}_k f\right|\right|_{\infty,k} \leq \left|\left|f\right|\right|_{1,k}$$

It is well-known (see [3],[4],[5],[6]) that the deformed Hankel transform $\mathcal{F}_k$ satisfies the following properties. Its inverse formula is given by

$$f(x) = \int_{\mathbb{R}} \mathcal{F}_k f(\lambda) B_k(\lambda x) d\mu_k(\lambda).$$

The Plancherel formula states

$$\|\mathcal{F}_k f\| = \left\|f_{2,k}\right\|$$

From [9], we have

$$\mathcal{F}_k(\mathbb{L}_k^r f)(\lambda) = (-1)^r |\lambda|^r \mathcal{F}_k f(\lambda), \qquad r \in \mathbb{N},$$

where $\mathbb{L}_k^r f = \mathbb{L}_k^r(\mathbb{L}_k^{r-1} f)$ and $\mathbb{L}_k^0 f = f$. The generalized translation operator T_y^k , verifies

$$\mathcal{F}_k(T_y^k f)(\lambda) = B_k(\lambda y)\mathcal{F}_k f(\lambda)$$

and we have $\left\|T_y^k f\right\|_{k,p} \leq A_k \|f\|_{k,p}$ for all $1 \leq p \leq \infty$ and $y \in \mathbb{R}$. Let $\mathcal{W}_{2,k}^m$ be the Sobolev space constructed by the $\mathbb{L}_k$ operator that is

$$\mathcal{W}_{2,k}^m := \{f \in L^2(d\mu_k) : \mathbb{L}_k^j f \in L^2(d\mu_k),\ j = 1, 2, \ldots, m\},$$

where $\mathbb{L}_k^j f = \mathbb{L}_k(\mathbb{L}_k^{j-1} f)$ and $\mathbb{L}_k^0 f = f$. Now we define the finite differences of order $m \in \mathbb{N}$ and step $h > 0$ by

$$\Delta_h^m f(\lambda) = (T_h^k - I)^m f(\lambda),$$

where I denotes the unit operator. Let $f \in L^2(d\mu_k)$ and $\delta > 0$. Then the generalized modulus of smoothness is defined by

$$w_m(f, \delta)_{2,k} = \sup_{0<h\leq\delta} \left\|\Delta_h^m f\right\|_{2,k}.$$

2 Results and Discussion

The main result of this talk is the following theorem, of equivalence between the K-functional and the modulus of smoothness:

Theorem 1. *There are two positive constants $c_1 = c(m,k)$ and $c_2 = c(m,k)$ such that*

$$c_1\, w_m(f,\delta)_{2,k} \;\le\; K_m(f,\delta^m)_{2,k} \;\le\; c_2 w_m(f,\delta)_{2,k}$$

where c_1 and c_2 depend on m and k.

This result is a generalization of a similar result for the classical Hankel transform, such that for $k = 0$ we obtain the classical Hankel transform.

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New Estimtes For The Fourier Transform In The Space $L^2(\mathbb{R}^n)$

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Abstract:
In this paper, we prove new estimates presented for the integral $\int_{|t|\geq N} |\widehat{f}(t)|^2 dt$ where $\widehat{f}$ stands for the Fourier transform of f and $N \geq 1$, in the space $L^2(\mathbb{R}^n)$ characterized by the generalized modulus of continuity of the k th order constructed with the help of the generalized spherical mean operator.

Keywords: Fourier Transform, Generalized derivatives, Spherical mean operator, Continuity modulus.

1 Introduction

In [2], Abilov et al. proved new estimates for the Fourier transform in the space $L^2(\mathbb{R})$ on certain classses of functions characterized by the generalized continuity modulus for these estimates, using a Steklov function.
In this paper, we prove the generalization of Abilov's results [2] in the Fourier transform for multivariable functions on $\mathbb{R}^n$. For this pupose, we use spherical mean operator in the place of Steklov function.

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Mathematical Model Of Tumor Growth

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Abstract:

Le cancer est une maladie complexe avec diverses caracteéristiques,

telles que la croissance rapide de cellules anormales et la capacité de se propager à d'autres organes.

On s'intéresse dans notre étude à la nécessité de comprendre comment les cancers se développent et comment les traitements peuvent être adaptés à chaque patient. On utilise les modèles mathématiques pour comprendre la croissance des cellules cancéreuses et pour simuler le taux de croissance des cellules cancéreuses. On se concentrent principalement sur la classe des cellules tumorales basées sur la structure d'âge ou poids. L'importance de ces modèles mathématiques c'est d'étudier la croissance des cellules tumorales et présenter une nouvelle approche basée sur des équations différentielles ordinaires et le modèle de Gompertz. prenant en compte à la fois l'âge et le poids comme variable indépendantes.

Keywords: Croissance tumorale, Modèles mathématiques, Les Modèles à compartiments. .

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Stabilization of a class of second order semilinear systems

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Abstract:
In this work we study strong and exponential stabilization of the following system

$$\begin{cases} \partial_{tt}y(t) + Ay(t) = u(t)B\partial_t y(t) \\ y_0 = y(0), \qquad y_1 = \partial_t y(0) \end{cases} \tag{1}$$

where A is a strictly positive and self-adjoint operator on a Hilbert space H endowed with the inner product $\langle .,. \rangle$ and the corresponding norm $\| \|_H$, the control operator $B : H \to H$ is nonlinear and bounded, $u(.)$ denotes the control function, and $X = V \times H$ is the state space where V endowed with the norm $\| \|_V$. We study the exponential and strong stabilization of bilinear second order systems, where the operator B does not necessarily commute with A. Our approach is based on one hand on the fact that A is strictly positive which is an important assumption that allows the uniquness of the solution and on the other hand on the choice of the control $u(t)$ which satisfies the dissipating energy inequality taking into account the fact that B verifies the observability inequality which make that the solution of the system converges to zero as $t \longrightarrow +\infty$.

Keywords: Second-order systems, bilinear systems, time delay, strong stabilization, exponential stabilization, wave equation, beam equation .

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Solution of Generalized Two-Dimensional Nonlinear Benjamin-Bona-Mahony-Burgers Equation Using Moving Least Square Method

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Abstract:
We apply the moving least square(MLS) method to approximate solution of the generalized 2-D nonlinear Benjamin–Bona–Mahony–Burgers (BBMB) equation. Formulation of MLS method is implemented of the generalized BBMB equation. The analysis errors of the proposed method is provided. Some numerical results are given and compared with analytical solutions to demonstrate the validity and efficiency of the proposed technique. The proposed method has excellent agreement to solve the generalized 2-D nonlinear BBMB equation on rectangular or non rectangular domain.

Keywords: Moving least square method, Nonlinear Benjamin–Bona–Mahony–Burgers (BBMB) equation, Analysis errors.

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A numerical scheme for solving boundary value problem of multi-order fractional differential equations

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Abstract:

In this work, a numerical method based on the Picard successive iteration technique in conjunction with the composite Simpson algorithm is developed to solve numerically the multi-order linear and nonlinear fractional differential equations. Our computational approach has ability to reduce the fractional problem into an integral equation. The derivatives are understood in the Riemann-Liouville sense. Next, discuss the analytic questions of existence and uniqueness of the exact solutions using a fixed point theorem and we investigate how the solutions depend on the some given assumptions. The efficiency and accuracy of the proposed result is examined by taking various test examples.

Keywords: Multi-term fractional differential equation, Riemann-Liouville derivatives, Banach's fixed point theorem, approximate solutions.

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Simulation numérique des écoulements de fluides viscoélastiques par la méthode des éléments finis

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Abstract:

The Navier–Stokes equations are fundamental in fluid mechanics. The finite element method has become a popular method for the solution of the Navier-Stokes equations. The numerical simulation of viscoelastic flow problems is a difficult and expensive task.. Most of the difficulties are numerical, improper boundary conditions, hyperbolic nature of the equations, instability of the classical FEM technique to deal with singularities, and the loss of convergence of almost all numerical schemes for high values of the Weissenberg number. The aim of this work is to study a finite element approximation for the Phan-Thien-Tanner viscoelastic fluid flow, which include a rheological Oldroyd-B model. We consider a decoupled method for solving PTT viscoelastic fluid flow. The method consists to solve alternatively a Stokes-like problem by weighted least-squares (WLS) finite element method, and the constitutive equation by streamline upwind Petrov–Galerkin (SUPG) method.

Key words : The Navier–Stokes equations, P-T-T model, finite element method, constitutive equation, Viscoelastic flows

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Physics Informed Neural Networks For One And Two Dimensional Reaction–Diffusion Brusselator System

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Abstract:
The main of this paper is to use a machine learning technique, called physics-informed neural network (PINN), to solve a 1D and 2D nonlinear rigid reaction-diffusion Brusselator system. PINN has succeeded in many scientific and engineering disciplines by encoding physical laws into the loss functions of the neural network, so that the network not only conforms to measurements, initial and boundaries, but also satisfies the governing equations. The use of PINN for the Brusselator system is still in its infancy, with many questions to be resolved, for example, the most appropriate form of physical laws. The performance of the method is verified by solving one-dimensional and two-dimensional test problems and comparing the results with those from numerical or analytical approaches. Validation of results is examined in terms of L^2 relative error. The result showed that the PINN performed well in producing good accuracy on benchmark problems.

Keywords: Physics-Informed Neural Network, reaction–diffusion Brusselator system, Residual-based Adaptive Refinement.

1 Introduction to Physics-Informed Neural Network

In recent years, deep learning methods have achieved unprecedented success in various application areas such as computer vision, speech recognition, natural language processing and audio recognition. Currently, researchers are looking into how to use these methods in solving partial differential equations (PDE). In particular, Raissi et al. [1] have proposed physics-informed neural networks (PINNs) to help solve PDE's and data-driven discovery.

PINNs can solve differential equations expressed, in the most general form, like:

$$\begin{cases} \mathcal{F}(u(z);\gamma) & = f(z) \qquad z \in \Omega, \\ \mathcal{B}(u(z)) & = g(z) \qquad z \in \partial\Omega \end{cases} \tag{1}$$

defined on the domain $\Omega \subset \mathbb{R}^d$ with the boundary $\partial\Omega$.
Where $z := [x_1, ..., x_{d-1}; t]$ indicates the space-time coordinate vector, u represents the unknown solution, γ are the parameters related to the physics, f is the function identifying the data of the problem and $\mathcal{F}$ is the non linear differential operator. Finally, since the initial condition can actually be considered as a type of Dirichlet boundary condition on the spatio-temporal domain, it is possible to denote $\mathcal{B}$ as the operator indicating arbitrary initial or boundary conditions related to the problem and g the boundary function. Indeed, the boundary conditions can be Dirichlet, Neumann, Robin, or periodic boundary conditions.
In the PINN methodology, $u(z)$ is computationally predicted by a Neural Network (NN), parametrized by a set of parameters θ, giving rise to an approximation:

$$\hat{u}_\theta(z) \approx u(z);$$

where $(\hat{\cdot})_\theta$ denotes a NN approximation realized with θ.
In such a context, the NN must learn to approximate the differential equations through finding θ that define the NN by minimizing a loss function that depends on the differential equation $\mathcal{L}_\mathcal{F}$, the boundary conditions $\mathcal{L}_\mathcal{B}$, and eventually some known data $\mathcal{L}_{data}$, each of them adequately weighted:

$$\theta^* = argmin_\theta(\omega_\mathcal{F}\mathcal{L}_\mathcal{F}(\theta) + \omega_\mathcal{B}\mathcal{L}_\mathcal{B}(\theta) + \omega_d\mathcal{L}_{data}(\theta))$$

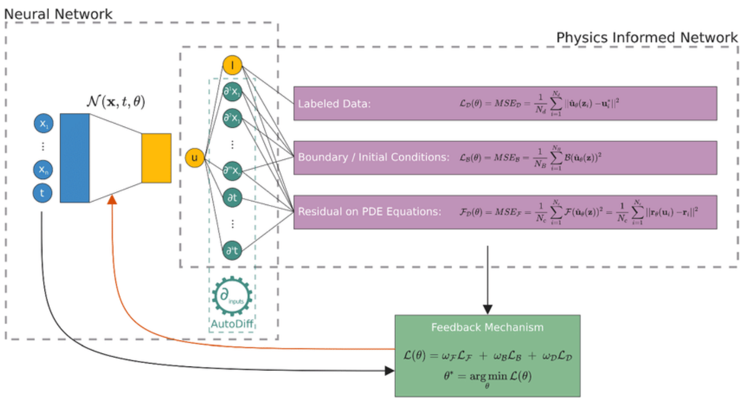

Figure 1: PINN's building blocks

Let consider the non-linear partial differential equations corresponding to the two-dimensional reaction–diffusion Brusselator system is given by [3]

$$\begin{cases} \dfrac{\partial u}{\partial t} &= \mu\Delta u + u^2 v - (\epsilon+1)u + \beta, \\ \dfrac{\partial v}{\partial t} &= \mu\Delta v - u^2 v + \epsilon u. \end{cases} \tag{2}$$

with Dirichlet and homogeneous Neumann boundary conditions

$$u(x,y,t) = f_1(x,y,t), v(x,y,t) = f_2(x,y,t), u_x(x,y,t) = 0, v_y(x,y,t) = 0, (x,y) \in \partial\Omega$$

and initial conditions $u(x,y,0) = g_1(x,y), v(x,y,0) = g_2(x,y), (x,y) \in \Omega$

Where $u(x,y,t)$ and $v(x,y,t)$ represent dimensionless concentrations of two reactants, β, ϵ constant concentrations of two input chemicals, μ is constant diffusion coefficient, f_1, f_2, g_1 and g_2 are known functions, Ω, $\partial\Omega$ interior and boundary of the domain, and Δ is the Laplacian operator.
There is very little literature available on the numerical solution of the mentioned model. For example, in [2] Sirajul Haq et al. proposed a numerical method based on combined Lucas and Fibonacci polynomials. Jiwari et al. [3] used a combined cubic B-spline method with RK4 scheme for two-dimensional system. Compared to previous works, we will evaluate in this contribution the performance of PINN in solving stiff Brusselator problems.

2 Results

Here, we give an efficiency of the PINN by solving some test problem. Consider 2 with the following initial and Neumann boundary conditions

$$\begin{aligned} &u(x,y,0) = 2 + 0.25y,\ v(x,y,0) = 1 + 0.8x. \\ &u_x(0,y,t) = u_x(1,y,t) = 0, \\ &v_y(x,0,t) = v_y(x,1,t) = 0, \qquad t > 0. \end{aligned}$$

Computations are carried out with the parameters $\epsilon = 1$, $\beta = 2$ and $\mu = 0.002$ over the domain $(x,y) \in [0,1]^2$ and $t \in [0,10]$. We compare our results with the results in [2] and [3], the solution is computed for time T varied from 0 to 10 and shown in Table 1 and 2. It is obvious from the tables that our proposed technique produced same results as recorded in [2] and [3].

T	Our PINN	[2]	[3]
1.0	u=2.3452 v=0.4166	u=2.3457 v=0.4162	u=2.3454 v=0.4163
2.0	u=2.0956 v=0.4689	u=2.0953 v=0.4688	u=2.0952 v=0.4689
7.0	u=2.0001 v=0.5000	u=2.0000 v=0.5000	u=2.0001 v=0.4999
9.0	u=2.0000 v=0.5000	u=2.0000 v=0.5000	u=2.0000 v=0.5000

Table 1: Approximate solution of example 3 at point (0.2, 0.2)

T	Our PINN	[2]	[3]
1.0	u=2.6052 v=0.3694	u=2.6323 v=0.3656	u=2.6069 v=0.3693
2.0	u=2.1764 v=0.4478	u=2.1839 v=0.4459	u=2.1757 v=0.4478
7.0	u=1.9996 v=0.5000	u=2.0011 v=0.49997	u=2.0001 v=0.4999
9.0	u=2.0000 v=0.5000	u=2.0000 v=0.5000	u=2.0000 v=0.5000

Table 2: Approximate solution of example 3 at point (0.8, 0.9)

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Dini Clifford Lipschitz Functions for the first Hankel-Clifford Transform

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Abstract. In this paper, using a generalized translation operator, we obtain an analog of Titchmarsh's and Younis's theorems for the first Hankel-Clifford transform on the half line for functions satisfying the Dini-Clifford Lipschitz condition in the space $L^2\left((0,+\infty), x^{\mu}\right)$, where $\mu \geq 0$.

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STABILITY ANALYSIS OF A SPATIALLY AND SIZE-STRUCTURED POPULATION MODEL WITH UNBOUNDED BIRTH PROCESS

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Abstract:
In this talk, we consider a spatially and size structured population model with unbounded birth process. Firstly, the model is transformed into a closed-loop system, and hence the well-posedness is established by using the feedback theory of regular linear systems. Moreover, the solution to the resulting closed-loop system is given by a perturbed semigroup. Secondly, we give a condition on birth and death rates in such a way that the solution decays exponentially. To do this, we show that the semigroup solution is positive and hence we derive a characterization of exponential stability due to the technique tools of positive semigroups.

Keywords: dynamic population, regular linear system, exponential stability.

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Necas Identity For The Biharmonic Operator In Lipschitz Domain

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Abstract:
We derive an identity, that go back to Nečas, for fourth order elliptic operators in smooth domains (see [4]), for the biharmonic operator in Lipschitz domain. This identity is useful in the control theory (see, [[2], [1], [6] and [5]).

Keywords: Lipschitz domain, biharmonic operator, Nečas identity, Hilbert space method.

1 Introduction

Let $\Omega \subset \mathbb{R}^d, d \geq 2$, be a bounded Lipschitz domain, let $\partial\Omega$ be its boundary and let $\overline{\Omega}$ be its closure. Let us consider for f defined on Ω, the Dirichlet problem for biharmonic equation

$$\begin{cases} \Delta^2 u = f & (\Omega) \\ u = \partial_\nu u = 0 & (\partial\Omega), \end{cases} \tag{1}$$

where Δ is the Laplacian. If $f \in H^{-2}(\Omega)$, it is well known that the problem (1) has a unique variational solution in $H_0^2(\Omega)$ ([7] and [9]) and if $f \in L^2(\Omega)$, and if Ω is suffisantly smooth, this solution is in $H^4(\Omega) \cap H_0^2(\Omega)$ (1) according to the shift theorem (see [7], [9], [4]).

The problem (1) when Ω is a lipschitz domain and $f \in L^2(\Omega)$ had attracted significant research attention.

2 Literature Review

We start with the work of J. Nečas [4] making use of the following identity [4], which for the bilaplacian and Ω smooth reads:

$$\begin{aligned}\int_{\partial\Omega}(m(x),\nu(x))_{\mathbb{R}^d}|(\Delta u)_{|\partial\Omega}|^2 d\sigma(x) &= \sum_{i=1}^{d}\int_{\Omega}\Delta m_i.\partial_i u.\Delta u dx \\ +2\sum_{i,j=1}^{d}\int_{\Omega}\partial_j m_i.\partial_{ij}^2 u.\Delta u dx &- \frac{1}{2}\int_{\Omega}\operatorname{div}(m)|\Delta u|^2 dx \\ &- \frac{1}{2}\int_{\Omega}(m,\nabla u)_{\mathbb{R}^d} f dx,\end{aligned} \tag{2}$$

where $m \in (\mathcal{C}^\infty(\mathbb{R}^d))^d$ is a vector field, ∇ is the gradient operator, div is the divergence operator, ∂_ν is the normal derivative operator associated to Ω with exterior normal ν, and $(\cdot,\cdot)_{\mathbb{R}^d}$ denotes the inner product on $\mathbb{R}^d$. Based on an approximation process of Lipschitz domains by smooth domains, and using Rellich identity (2), Nečas proved that the trace of Δu, where u is the solution of problem (1) on a bounded Lipschitz domain, is square integrable on the boundary of the domain. Then he showed, in the theorem 4.1 [4], that $\partial_\nu \Delta u \in H^{-1}(\partial\Omega)$. These results show that we can use the duality to find a very weak solution of the problem :

$$\begin{cases}\Delta^2 v = 0 & \text{in} \quad \Omega \\ v = g^0 & \text{on} \quad \partial\Omega, \\ -\partial_\nu v = g^1 & \text{on} \quad \partial\Omega.\end{cases} \tag{3}$$

3 Results and Discussion

Our major contribution is to prove the validity of the identity (2) in the case where $f \in H^{-1}(\Omega)$, and when proving this result, we established that $\Delta u \in H^1(\Omega)$ for all $u \in H_0^2(\Omega)$ solution of the problem (1), which will allow us to improve the result of Nečas by proving that $\Delta u_{|\partial\Omega} \in H^{1/2}(\partial\Omega)$.

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Tikhonov regularization problem and Toeplitz operators for the Laguerre-Bessel wavelet transform

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Abstract:

The main crux of this work is to study the wavelet transform associated with the Laguerre-Bessel operator and to give some results related to this transform as Parseval's formula and reproducing property, next using reproducing kernel theory we give an integral representation of the extremal function associated to this transform. In the last we define a class of pseudo-differential operator $T_{u,v}(\sigma)$ called localization operator wich depend on a symbol σ and two functions u and v, we give a criteria in terms of the symbol σ for its boundedness and compactness, we also show that these operators belongs to the Schatten-Von Neumann class S^p for all $p \in [1;+\infty]$ and we give a trace formula.

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Generalized Poisson transform of an L^p-function over the Shilov boundary of the symmetric domains of non-tube type

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Abstract:
Let $\Omega = G/K$ be an irreducible bounded symmetric domain of non-tube type of rank r in $\mathbb{C}^n$ and $\mathcal{B}(\mathcal{S})$ be the space of hyperfunctions on the Shilov boundary $\mathcal{S}$ of Ω.
The purpose of this talk is to give a necessary and sufficient condition on the generalized Poisson transform $\mathcal{P}_{\lambda,\nu}$ of an element f in the space $\mathcal{B}(\mathcal{S})$ for f to be in $L^p(\mathcal{S})$, $(1 < p < \infty)$ or f to be a Borel measure on the Shilov boundary.

Keywords: Bounded symmetric domain, Shilov boundary, homogeneous vector bundles, generalized Hua operator, generalized Poisson transform.

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On the Continuous embeddings between the fractional Hajłasz-Orlicz-Sobolev spaces

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Abstract :

Let G be an Orlicz function and let α, β, s be positive real numbers. Under certain conditions on the Orlicz function G, we establish some continuous embeddings results between the fractional order Orlicz-Sobolev spaces defined on metric-measure spaces $W_s^{\alpha,G}(X, d, \mu)$ and the fractional Hajłasz-Orlicz-Sobolev spaces $M^{\beta,G}(X, d, \mu)$.

Keywords : Sobolev spaces, Orlicz spaces, Hajłasz-Orlicz-Sobolev spaces, Fractional spaces, Metric-measure spaces, Sobolev embeddings.

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Nonhomogeneous Dirichlet Problems For Fractional Elliptic Equations With Variable Exponent.

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Abstract:
In this work, we are concerned with the existence of solutions for the nonhomogeneous Dirichlet problem

$$\begin{cases} \Big(- \Delta_{p(x,.)} \Big)^s u(x) = \mathcal{B}(x,u) & \text{on } \Omega, \\ u = g & \text{in } \mathbb{R}^N \setminus \Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^N$ is a smooth bounded domain, $\Big(- \Delta_{p(x,.)} \Big)^s$ is the fractional $p(x,.)$-Laplacian, with $p : \mathbb{R}^{2N} \longrightarrow (1;\infty)$ is bounded, continuous and symmetric function. $\mathcal{B} : \Omega \times \mathbb{R} \longrightarrow \mathbb{R}$ is a Carathéodory function. The proof of our existence result is established by employing a combination of fixed point argument and Calderón-Zygmund type regularity result for the nonhomogeneous fractional Poisson equation.

Keywords: Fractional $p(x.)$-Laplacian operator,Fractional Sobolev Spaces with variable exponent, Nonhomogeneous nonlocal problems.

1 Introduction

In this paper we are interested in the existence of weak solutions for the following problem

$$\begin{cases} \Big(- \Delta_{p(x,.)} \Big)^s u(x) = \mathcal{B}(x,u) & \text{on } \Omega, \\ u = g & \text{in } \mathbb{R}^N \setminus \Omega, \end{cases} \tag{1}$$

where $\Omega \subset \mathbb{R}^N$ is a smooth bounded domain, $s \in (0,1)$, $\mathcal{B} : \Omega \times \mathbb{R} \mapsto \mathbb{R}$ is a Carathéodory function, g is a given boundary data, and $\Big(-\Delta_{p(x,.)}\Big)^s$ is the fractional $p(x,.)$-Laplacian operator defined as

$$(-\Delta_{p(x,.)})^s u(x) = p.v \int_{\mathbb{R}^N} \frac{|u(x)-u(y)|^{p(x,y)-2}(u(x)-u(y))}{|x-y|^{N+sp(x,y)}} dy,$$

for all $x \in \mathbb{R}^N$, where $p : \overline{\Omega} \times \overline{\Omega} \mapsto (1,\infty)$ is bounded, continuous and symmetric function.

In recent years, there has been a growing interest in investigating nonlinear problems that involve the fractional $p(x,.)$-Laplacian in fractional Sobolev spaces with variable exponents $W^{s,q(.),p(.,.)}(\Omega)$, see [1, 2, 3, 4, 5, 6] and the references therein.

Kaufmann, Rossi, and Vidal [7] were the first to introduce results on fractional Sobolev spaces with variable exponents $W^{s,q(.),p(.,.)}(\Omega)$ and the fractional $p(x,.)$-Laplacian, in which they established compact embedding theorems of these spaces into variable exponent Lebesgue spaces. As an application, they also showed the existence and uniqueness of solutions for nonlocal problems that involve the fractional $p(x,.)$-Laplacian. In [6] Bahrouni and Rădulescu obtained some further qualitative properties of the fractional Sobolev space $W^{s,q(.),p(.,.)}(\Omega)$ and the fractional $p(x,.)$-Laplacian. Subsequently, the authors in [1] further refined the fractional Sobolev spaces with variable exponents introduced in [7] and established fundamental embeddings of these spaces.

Several interesting problems are included in Problem (1). For instance when the exponent p is constant and $g = 0$, Problem (1) becomes

$$\begin{cases} \big(-\Delta_p\big)^s u(x) = \mathcal{B}(x,u) & \text{on } \Omega, \\ u = 0 & \text{in } \mathbb{R}^N \setminus \Omega, \end{cases} \tag{2}$$

where $\Big(-\Delta_p\Big)^s$ is the well known fractional p-Laplacian operator; see [8] and the references given there.

Problem (1) can be seen as the fractional form of the following problem

$$\begin{cases} -\Delta_{p(.)} u + \mathcal{B}(x,u) = 0 & \text{in } \Omega, \\ u = g \text{ on } \partial\Omega. \end{cases} \tag{3}$$

Let us also point out that in [10], using approximation argument, the authors prove that for every continuous function g on the boundary $\partial\Omega$ there exists a unique continuous solution of Problem (3). Notice that our situation is different from [10], because of the presence of a nonlocal operator, in the sens that the value of $(-\Delta_{p(x,.)})^s u(x)$ at any point $x \in \Omega$ depends not only on the value of u on Ω, but actually on its value on all of $\mathbb{R}^N$.

Our interest in Problem (1) has originated from the paper [11], in which A.Baalal proves the existence and uniqueness of a weak solution to the Dirichlet problem

$$\begin{cases} \mathcal{L}u := \sum_j \frac{\partial}{\partial x_j}\Big(\sum_i \alpha_{i,j}\frac{\partial u}{\partial x_i} + \delta_i u\Big) = \mathcal{B}(., u, \nabla u) \quad \text{in } \Omega, \\ \qquad\qquad\qquad\qquad\qquad\qquad u = g \text{ on } \partial\Omega. \end{cases} \tag{4}$$

Through the change of unknown $v = u - \tilde{g}$, where $\tilde{g}$ is an extension of g to Ω, the nonhomogeneous Dirichlet problem given in (4) reduced into a homogeneous Dirichlet problem. However the nonlinear case is quite different, in our case, an analogous change of unknown function leads to consider the following operator $v \longrightarrow \Big(-\Delta_{p(x,.)}\Big)^s (v + \tilde{g})$. This operator turns out to be harder to handle than the fractional $p(x,.)$-Laplacian, and for that matter, we will deal directly with the nonhomogeneous Dirichlet Problem (1).

The proof of our existence result relies on a combination of fixed point arguments, in the spirit of [11], and a Calderón-Zygmund type result for the fractional nonhomogeneous Poisson equation associated to Problem (1),

$$\begin{cases} \Big(-\Delta_{p(x,.)}\Big)^s u(x) = f \quad \text{on } \Omega, \\ \qquad\qquad u = g \quad \text{in } \mathbb{R}^N \setminus \Omega, \end{cases} \tag{5}$$

We believe that this result has independent value and can be applied in various other settings. Specifically, these results can be considered as a self-contained and independent part of our work.

Having at hand regularity result for (5) and inspired by [11], we develop a fixed point argument to obtain a solution to (1). Note that, due to the nonlocality and nonlinearity of the operator and the nonhomogeneous Dirichlet condition, the approach of [11] has to be adapted significantly. This paper appears to be the first to investigate the existence of weak solutions to fractional $p(x,\cdot)$-Laplacian problems with nonhomogeneous Dirichlet conditions.

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International Conference on Recent Advances in Partial Differential Equations and Applications (ICRAPDEA'23)

July 27-28, 2023, Faculty of Sciences Casablanca, Hassan II University, Casablanca Morocco

Local integrability of $G(\cdot)$-superharmonic functions in Lebesgue and Musielak-Orlicz spaces

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Abstract In this talk, we study local integrability properties of superharmonic functions related to partial differential equations with Musielak-Orlicz growth conditions in Lebesgue and Musielak-Orlicz spaces.

Keywords: $G(\cdot)$-superharmonic, Local integrability, Musielak-Orlicz growth, Generalized Φ-function.

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